Renaissance of Strong Field Physics

Vacuum Physics at Critical (Planck) Acceleration

presented by **Jan Rafelski** at:

50 Cracov School of Theoretical Physics

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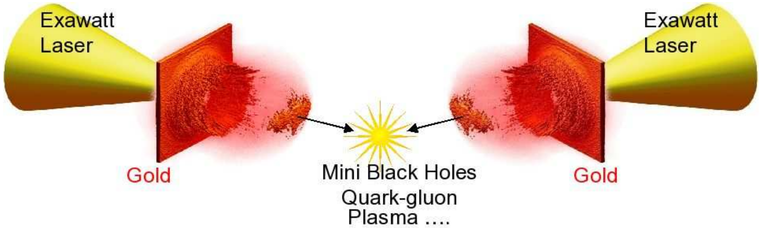
Credits to: **Lance Labun** and Yaron Hadad
The University of Arizona

Overview

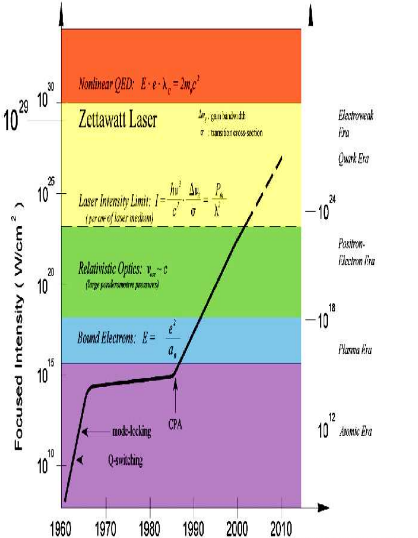
1. New beam in physics: Laser pulse
2. Acceleration: Mach, Planck, Aether
3. The Vacuum as source of laws of physics
4. Back to classics: LAD equations
5. Radiation Reaction Dominance



From PRL, S.A. Bulanov et al.

Laser Pulse

– A new tool to study vacuum and high acceleration physics



(Tajima,Mourou,2002)

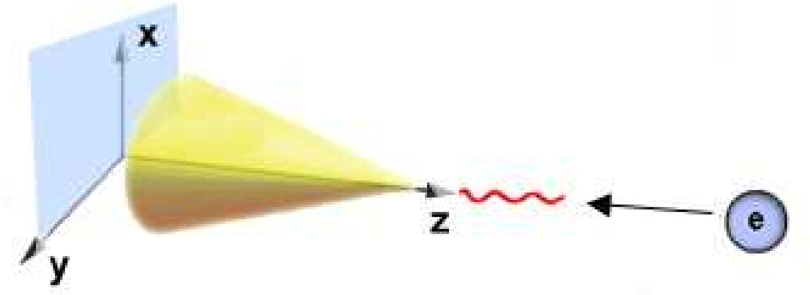
Laser Pulse – Energy

Coherent Light: $10^{23} \times 1$ eV photons $\equiv$ 16 kJ

Comparison

7TeV proton: $\equiv 0.710^{-10}$ fraction of pulse

$M_{\rm Planck} = 1.22 \times 10^{28}$ eV, 100,000 more



Laser Pulse – Parameters

Present Limits of technology:

Energy: 1 kJ aiming at 10kJ

Pulse length - 10 $\times$ wavelength aiming at 2-3

$$\lambda \equiv \frac{hc}{1\mathrm{eV}} \equiv 1.25\mu,\ 10\lambda \equiv 12.5\frac{\mu}{c} = 40\mathrm{fs}.$$

$a_0 \equiv |\vec{A}|/mc^2 = 100$ aiming at 1000–10,000

To reach Plank scale:

Energy: a factor 1000 000.

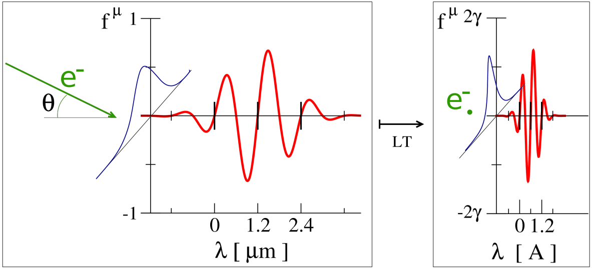
Force: $a_o = mc^2/1\mathrm{eV} = 500,000$

Technology needs another factor 1000

– but difficulty grows with $a_0^2 \propto \vec{E} \times \vec{B}; \vec{E}^{\,2}; \vec{B}^{\,2}$

Simple idea: Lorentz-boost.

Laser Pulse in Electron's Restframe



Force applied by a Gaussian photon pulse with $\gamma / \cos\theta = 2000$ – narrowed by $(\gamma \cos\theta)^{-1}$ in the longitudinal and $(\gamma \sin\theta)^{-1}$ in the transverse direction. The magnitude of the effect described by the Doppler-shift:

$$\omega \to \omega' = \gamma(\omega + \vec{v} \cdot nk)$$

Critical Pulse

In electron's rest frame: a pulse of 10^{23} photons approaching with energy 2γ eV.
For a practical electron energy of 50 GeV, $\gamma = 100,000$ a laser pulse of $a_0 = 1000$ is a Planck energy pulse in rest frame of the electron: its energy is at Planck scale and its acceleration force 20 times above the critical (unit) acceleration.

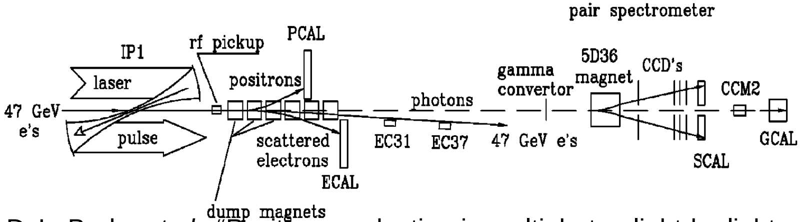
Probing unit acceleration possible today

SLAC'95 experiment — *Proof of Principle*

$$p_e^0 = 46.6\ \text{GeV}, \quad a_0 = 0.4, \quad \left|\frac{du^\alpha}{d\tau}\right| = .073[m_e]\ \text{(Peak)}$$

Multi-photon processes observed:

- Nonlinear Compton scattering
- Breit-Wheeler electron-positron pairs



- D. L. Burke *et al.*, "Positron production in multiphoton light-by-light scattering," Phys. Rev. Lett. **79**, 1626 (1997)
- C. Bamber *et al.*, "Studies of nonlinear QED in collisions of 46.6 GeV electrons with intense laser pulses" Phys. Rev. D **60**, 092004 (1999).

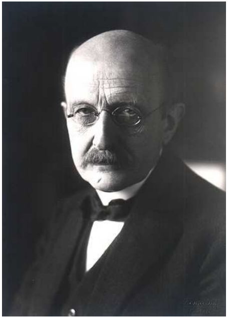
Inertia and Mach's Principle: ca 1895

- To define acceleration another inertial reference frame, such as matter frame of the Universe, CBM-frame, the quantum vacuum, the geometric manifold - space in Einstein gravity is needed

- Einstein got rid of dynamic acceleration: point masses are in a free fall; rotating solutions and frame drag remains – "Mach's Principle" re-introduced by Einstein in 1918
- (linear) Acceleration arises in presence of quantum matter and vnon-geometric e.g. quantum forces that create a rigid extended material body.

Newton's absolute space is dead. Long Live Mach's Principle.
Just that in all microscopic theory we build there is no Mach.

1899: Planck units



$$h/k_B = a = 0.4818 \cdot 10^{-10} [\text{sec} \times \text{Celsiusgrad}]$$
$$h = b = 6.885 \cdot 10^{-27} \left[\frac{\text{cm}^2\text{gr}}{\text{sec}}\right]$$
$$c = c = 3.00 \cdot 10^{10} \left[\frac{\text{cm}}{\text{sec}}\right]$$
$$G = f = 6.685 \cdot 10^{-8} \left[\frac{\text{cm}^3}{\text{gr. sec}^2}\right]^1.$$

Wählt man nun die »natürlichen Einheiten« so, dass in dem neuen Maasssystem jede der vorstehenden vier Constanten den Werth 1 annimmt, so erhält man als Einheit der Länge die Grösse:

$$\sqrt{2\pi}\, L_{Pl} = \sqrt{\frac{bf}{c^3}} = 4.13 \cdot 10^{-33}\,\text{cm}, \mapsto \sqrt{2\pi}\, 1.62 \times 10^{-33}\ \text{cm}$$

als Einheit der Masse:

$$\sqrt{2\pi}\, M_{Pl} = \sqrt{\frac{bc}{f}} = 5.56 \cdot 10^{-5}\,\text{gr}, \mapsto \sqrt{2\pi}\, 2.18 \times 10^{-5}\ \text{g}$$

als Einheit der Zeit:

$$\sqrt{2\pi}\, t_{Pl} = \sqrt{\frac{bf}{c^5}} = 1.38 \cdot 10^{-43}\,\text{sec}, \mapsto \sqrt{2\pi}\, 5.40 \times 10^{-44}\ \text{s}$$

als Einheit der Temperatur:

$$\sqrt{2\pi}\, T_{Pl} = a\sqrt{\frac{c^5}{bf}} = 3.50 \cdot 10^{32}\,{}^\circ\text{Cels.} \mapsto \sqrt{2\pi}\, 1.42 \times 10^{32}\ \text{K}$$

Diese Grössen behalten ihre natürliche Bedeutung so lange bei, als die Gesetze der Gravitation, der Lichtfortpflanzung im Vacuum und die beiden Hauptsätze der Wärmetheorie in Gültigkeit bleiben, sie müssen also, von den verschiedensten Intelligenzen nach den verschiedensten Methoden gemessen, sich immer wieder als die nämlichen ergeben.

“These scales retain their natural meaning as long as the law of gravitation, the velocity of light in vacuum and the central equations of thermodynamics remain valid, and therefore they must always arise, among different intelligences employing different means of measuring.” *M. Planck, “Über irreversible Strahlungsvorgänge.” Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften zu Berlin* **5**, *440-480 (1899), (last page)*

1920: Einstein-Lorentz and the Vacuum (Aether)

Albert Einstein rejected æther as unobservable when formulating special relativity, but eventually changed his initial position, re-introducing what is referred to as the 'relativistically invariant' æther. In a letter to H.A. Lorentz of November 15, 1919, see page 2 in *Einstein and the Æther*, L. Kostro, Apeiron, Montreal (2000). he writes:

It would have been more correct if I had limited myself, in my earlier publications, to emphasizing only the non-existence of an æther velocity, instead of arguing the total non-existence of the æther, for I can see that with the word æther we say nothing else than that space has to be viewed as a carrier of physical qualities.

In a lecture published in May 1920 (given on 27 October 1920 at Reichs-Universität zu Leiden, addressing H. Lorentz), published in Berlin by Julius Springer, 1920, also in Einstein collected works: In conclusion:

. . . space is endowed with physical qualities; in this sense, therefore, there exists an æther. . . . But this æther may not be thought of as endowed with the quality characteristic of ponderable media, as (NOT) consisting of parts which may be tracked through time. The idea of motion may not be applied to it.

Acceleration – Physics Riddles

Framework of (electromagnetic) theory is incomplete:

Current procedure:

1) Inertial Force = Lorentz-force → get world line of particles =source of fields:

$$m_e \frac{du^\alpha}{d\tau} = -eF^{\alpha\beta}u_\beta \quad \rightarrow x^\alpha(\tau), u^\alpha(\tau) \quad \rightarrow j^\alpha$$

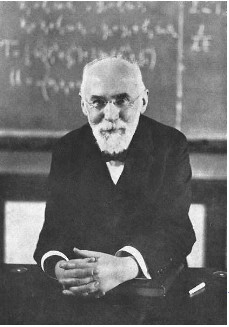
2) Source of Fields = Maxwell fields –> get fields, omitting radiated fields

$$\partial_\beta F^{\beta\alpha} = j^\alpha \quad \rightarrow F^{\beta\alpha}$$

3) Fields fix Lorentz force –> go to **1)**

As long as acceleration is small, radiation emitted can be incorporated as a perturbative additional force. For large acceleration this is a new source of resistance to acceleration.

Radiation reaction force



energy-momentum radiated to order e^2:

$$\frac{dp^\alpha}{d\tau} = -\frac{2e^2}{3} u^\alpha \frac{du^\beta}{d\tau}\frac{du_\beta}{d\tau}$$

Recognized and further developed among others by

← Lorentz Dirac →

<u>At unit acceleration</u> radiation impact on charged particle dynamics require an unknown nonperturbative extension of dynamics. Quantum theory (QED) is build on this limited frame work

Critical (Planck) Acceleration

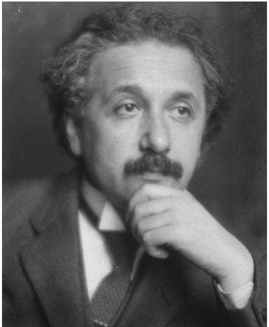
Planck mass scale corresponds to unit strength Newton force: for $m \to M_{\rm Pl}$:

$$f_{\rm grav} \simeq \frac{Gm^2}{r^2} \to \frac{\hbar c}{r^2} \quad \to 1\left[\frac{m^2c^3}{\hbar}\right] \quad \text{at} \quad r = \frac{\hbar}{mc}$$

Einstein's gravity is built upon Equivalence Principle: a relation of gravity to inertia(=the resistance to acceleration).

We study the same Planck scale physics when exposing particles to forces where acceleration is unit:

$$m\frac{du^\alpha}{d\tau} = f^\alpha \to 1^\alpha \quad \text{in natural units} \quad \left[\frac{m^2c^3}{\hbar}\right]$$



How to invent a better theory with acceleration

Laws of Physics originate in Quantum Vacuum Properties

Development of quantum physics leads to the recognition that vacuum fluctuations define laws of physics: Weinberg's effective theory picture.

- The 'quantum æther' is polarizable: Coulomb law is modified; E.A.Uehling, 1935
- New interactions (anomalies) such as light-light scattering arise considering the electron, positron vacuum zero-point energy; Euler, Kockel, Heisenberg (1930-36);
- Casimir notices that the photon vacuum zero point energy also induces a new force, referred today as Casimir force 1949
- Non-fundamental vacuum symmetry breaking particles possible: Goldstone Bosons '60-s
- 'Fundamental electro-weak theory is effective - model of EW interactions, 'current' masses as VEV Weinberg-Salam '70-s
- Color confinement and high-T deconfinement Quark-Gluon Plasma '80-s

QED Non-perturbative Vacuum

Vacuum state: $\hat{b}_e|0\rangle = \hat{d}_{\bar{e}}|0\rangle = 0$
e particle states and $\bar{e}$ antiparticle 'modes'

$$\hat{\Psi} = \sum_e \psi_e \hat{b}_e + \sum_{\bar{e}} \psi_{\bar{e}} \hat{d}^\dagger_{\bar{e}},$$

Charge operator introduced such that the Dirac 'sea' charge of positrons cancels charge of electrons (net zero charge fluctuations):

$$\hat{Q} = \int d^3x \frac{e}{2}\left[\Psi(x)^\dagger, \Psi(x)\right], \qquad \langle 0|\hat{Q}|0\rangle = \frac{e}{2}\left[\sum_e - \sum_{\bar{e}}\right] \to 0$$

In past nonperturbative computation of vacuum fluctuations for zero point energy carried out for four independent Dirac particle seas: electron, positron, spin $\pm$. See seminar lecture of Lance Labun.

$$\mathcal{E} = -\frac{1}{2V}\sum_{e,s}\varepsilon_e - \frac{1}{2V}\sum_{\bar{e},s}(-\varepsilon_{\bar{e}}) \to (g_{\rm Bos} - g_{\rm Ferm})M^4_{\rm Planck} + d_1\bar{m}^2 M^2_{\rm Pl} + \mathcal{L}(E, B)$$

Last nonperturbative term proportional to particle degeneracy, no constraint from charge and spin of the vacuum.

Euler-Heisenberg Z. Physik 98, 714 (1936) evaluate Dirac zero-point energy with (constant on scale of $\hbar/mc$) EM- fields, transform to action $\mathcal{E}(D,H) \to \mathcal{L}(E,B) = \mathcal{E} - ED$, $E \equiv \frac{\partial\mathcal{E}}{\partial D}$

$$\mathcal{L}(E,B) = -\frac{1}{8\pi^2}\int_0^\infty \frac{ds}{s^3}\mathrm{e}^{-m^2 s}\left[\frac{sE}{\tan sE}\,\frac{sB}{\tanh sB} - 1 + \frac{1}{3}(E^2 - B^2)s^2\right].$$

E, B relativistic definition in terms of invariants

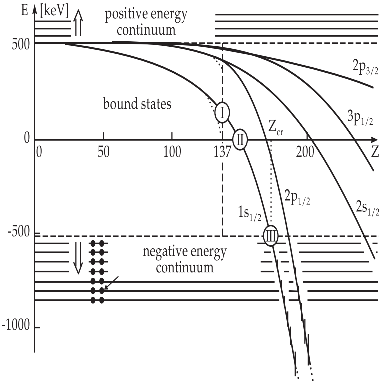
$$\mathcal{L}(E,B) \to \frac{2\alpha^2}{45m^4}\left[\left(\vec{E}^{\,2} - \vec{B}^{\,2}\right)^2 + 7(\vec{E}\cdot\vec{B})^2\right] = \mathrm{light-light\ scattering}$$

Schwinger 1951 notes importance of imaginary part due to zeros of $\tan sE$:

$$|\langle 0_+|0_-\rangle|^2 = e^{-2\mathcal{I}m\,\mathcal{L}}, \qquad 2\mathcal{I}m\,\mathcal{L} = \frac{\alpha^2}{\pi^2}\sum_{n=1}^{\infty} n^{-2} e^{-n\pi m^2/eE}$$

is the vacuum persistence probability in adiabatic switching on/off the E-field. For $E \to 0$ essential singularity.

High Z Atoms and QED Vacuum



Single Particle Dirac Equation

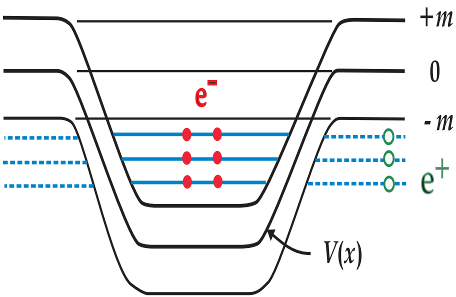
$$(\vec{\alpha}\cdot i\vec{\nabla}+\beta m+V(r))\Psi_n(\vec{r}) = E_n\Psi_n(\vec{r})$$

$$V(r) = \begin{cases} -\frac{Z\alpha}{r} & r > R_N \\ -\frac{3}{2}\frac{Z\alpha}{R_N} + \frac{r^2}{2}\frac{Z\alpha}{R_N^3} & r < R_N \end{cases}$$

The bound states drawn from one continuum move as function of $Z\alpha$ across into the other continuum.

References: The large volume of work from 1968-85 is reviewed in W. Greiner, B. Müller and JR "Quantum Electrodynamics of Strong Fields," (Springer Texts and Monographs in Physics, 1985), ISBN 3-540-13404-2.

Formation of Charged Vacuum

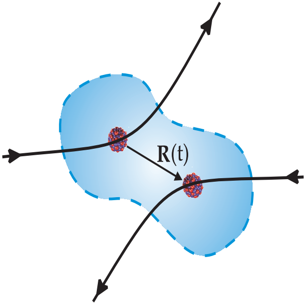


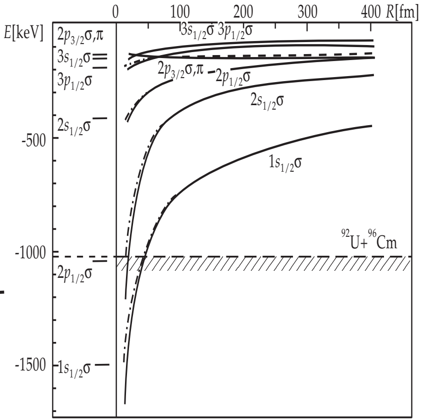
Pair production across a nearly constant field fills the additional ‘dived’ states available in the localized domain. There is real localized charge density in the vacuum. Formation of the charged vacuum ground state observable in positron emission.

Back-reaction of real vacuum charge and screening of the external field (Müller and JR, PRL34, 349 (1975)):

$$-\vec{\nabla}^2 V = \rho_N + \langle \rho_e \rangle, \qquad \langle \rho_e \rangle \simeq -\frac{e^2}{3\pi^2}(2mV + V^2)^{3/2}\Theta(-V - 2m)$$

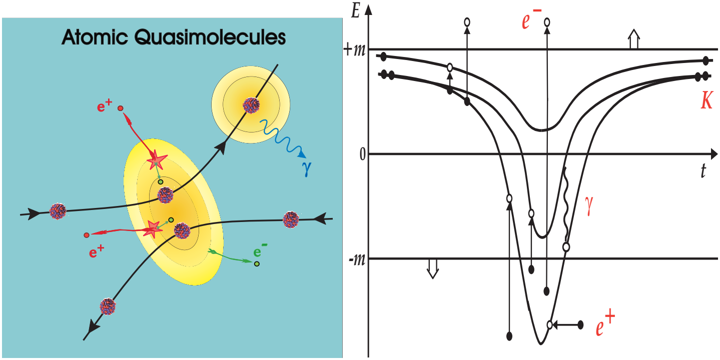
Experimental Effort (presently resurrected)





The velocity of electrons is near that of light, that of ions about c/10: Born-Oppenheimer quasi-molecular two-center orbitals. For $R < 30$ fm Supercritical σ-states.

Time Dependent Processes



The natural vacuum life span (vacuum resonance width) longer than typical duration of collision. The time dependence induces particle production process with energy extracted from collision: ‘assisted pair production’ (invented 1975, reinvented in 2008). Phys.Rev.D78:061701,2008 *Pair Prod. Beyond Schwinger Formula in t-dep. E-fields*. Too many pairs, not possible to identify which are relevant for study of vacuum structure

Need Strong Fields Without Heavy Ion Collisions

1. Supercritical fields in heavy ion collisions exist, but for too short a time, positron dynamically spread in energy and the total yield is small
2. Positron production from other time dependent processes offers a dominant background with a broad spectrum
3. Efforts to 'glue' heavy nuclei in grazing collisions resulted in nuclear contributions to particle production but no sharp lines

Today: Renewed effort under consideration. However, alternate technology can produce a new source of supercritical fields: femto second laser pulsed lasers today with $N_\gamma = 0.610^{22}$ photons and pulse length is 10-70 fs, focus diffraction limited at $(2\mu\mathrm{m})^3$. The energy density: $\epsilon \simeq 100\mathrm{J}/\mu\mathrm{m}^3 =$ $= 600\mathrm{MeV}/(10^{-10}\mathrm{m})^3$ is very high on atomic scale: pulse contains nearly $E = mc^2$ hydrogen gas energy.

LAD equations – to be specific

$$m_e\,\dot{u}^\mu = -\frac{e}{c}F^{\mu\nu}u_\nu\ , \qquad u^\mu = \dot{s}^\mu(\tau)\ , \qquad \partial^\alpha\partial_\alpha A^\mu = \frac{1}{\epsilon_0 c^2}\,j^\mu$$

$$j^\mu(x) = -ec\int d\tau\, u^\mu[s(\tau)]\,\delta^4[x - s(\tau)]$$

$$A^\mu_{\rm rad} = -\frac{e}{\epsilon_0 c}\int d\tau\, u^\mu[s(\tau)]\,G_+[x - s(\tau)]$$

$$-\frac{e}{c}\,F^{\mu\nu}_{\rm rad}\,u_\nu = \frac{e^2}{\epsilon_0 c}\int d\tau\, u_\nu(x)\,\left(u^\nu[s(\tau)]\,\partial^\mu - u^\mu[s(\tau)]\,\partial^\nu\right)\,G_+[x - x(\tau)]$$

$$G_\pm = \theta[\pm X_0]\,\delta[X^2]\ , \qquad X^\mu = x^\mu - x^{\mu\,\prime}$$

$$-\frac{e}{c}\,F^{\mu\nu}_{\rm rad}\,u_\nu = \frac{2e^2}{\epsilon_0 c}\int d\tau\, u_\nu\,\left(u^{\nu\,\prime}X^\mu - u^{\mu\,\prime}X^\nu\right)\,\frac{\partial G_+}{\partial X^2}$$

Expansion for far zone – Dirac 1938

$$X^\mu \approx \delta u^\mu - \frac{\delta^2}{2}\dot{u}^\mu + \frac{\delta^3}{6}\ddot{u}^\mu \pm \dots\,, \qquad u^{\mu\,\prime} \approx u^\mu - \delta\dot{u}^\mu + \frac{\delta^2}{2}\ddot{u}^\mu \pm \dots$$

$$\delta = t - \tau, \qquad X^2 \approx c^2\,\delta^2 \to \frac{\partial}{\partial X^2} = \frac{1}{2c^2\delta}\,\frac{\partial}{\partial\delta}$$

$$\begin{aligned} F^\mu_{\mathrm{rad}} &= \frac{e^2}{\epsilon_0 c}\int d\delta\,\frac{\partial G_+}{\partial\delta}\left(\frac{\delta}{2}\dot{u}^\mu - \frac{\delta^2}{3}\left[\ddot{u}^\mu + \frac{1}{c^2}\,\dot{u}^\eta\dot{u}_\eta u^\mu\right]\right) \\ &= -\frac{e^2}{2\epsilon_0 c^3}\dot{u}^\mu \int d\delta\,\frac{\Delta[\delta]}{|\delta|} + \frac{2e^2}{3\epsilon_0 c^3}\left[\ddot{u}^\mu + \frac{1}{c^2}\,\dot{u}^\eta\dot{u}_\eta u^\mu\right] \end{aligned}$$

$$m_r\,\dot{u}^\mu = -\frac{e}{c}F^{\mu\nu}u_\nu + F^\mu_{\mathrm{LAD}}, \quad m_r = m_e\left(1 + \frac{e^2}{2\epsilon_0 c^3}\int d\delta\,\frac{\Delta[\delta]}{|\delta|}\right)$$

$$F^\mu_{\mathrm{LAD}}u_\mu = \frac{2e^2}{3\epsilon_0 c^3}\left[\ddot{u}^\mu + \frac{1}{c^2}\,\dot{u}^\eta\dot{u}_\eta u^\mu\right]u_\mu = 0\,, \qquad u^2 = 1.$$

LAD → Landau-Lifshitz

LAD has conceptual problems (run-away solutions).

$$m_e \dot{u}^\mu = -\frac{e}{c} F^{\mu\nu} u_\nu + F^\mu_{\mathrm{LAD}} \qquad F^\mu_{\mathrm{LAD}} = \frac{2e^2}{3\epsilon_0 c^3} \left[\ddot{u}^\mu + \frac{1}{c^2} \dot{u}^\eta \dot{u}_\eta u^\mu \right]$$

Iterate using Lorentz force and its differential in LAD:

$$\begin{aligned} \ddot{u}^\mu &\rightarrow \frac{d}{d\tau}\left(-\frac{e}{m_e c} F^{\mu\nu} u_\nu\right) = -\frac{e}{m_e c}\left(\partial_\eta F^{\mu\nu} u_\nu u^\eta + F^{\mu\nu} \dot{u}_\nu\right) \\ &= -\frac{e}{m_e c}\left(\partial_\eta F^{\mu\nu} u_\nu u^\eta - \frac{e}{m_e c} F^{\mu\nu} F^\eta_\nu u_\eta\right) \end{aligned}$$

$$F^\mu_{\mathrm{LAD}} \simeq -\frac{2e^3}{3\epsilon_0 m_e c^4}\left(\partial_\eta F^{\mu\nu} u_\nu u^\eta - \frac{e}{m_e c} F^{\mu\nu} F^\eta_\nu u_\eta\right) + \frac{2e^4}{3\epsilon_0 m_e^2 c^7} F^{\eta\nu} F_{\eta\delta} u_\nu u^\delta u^\mu$$

This is equivalent to LAD only for weak acceleration. But like LAD it is a heuristic description of high acceleration domain and is often taken to be equally valid model of strong acceleration regime. No conceptual problems and is EXACTLY soluble: di Piazza 2009, Hadad et al 2010.

LAD → Caldirola

Caldirola notices that derivative terms comprise a nonlocality!

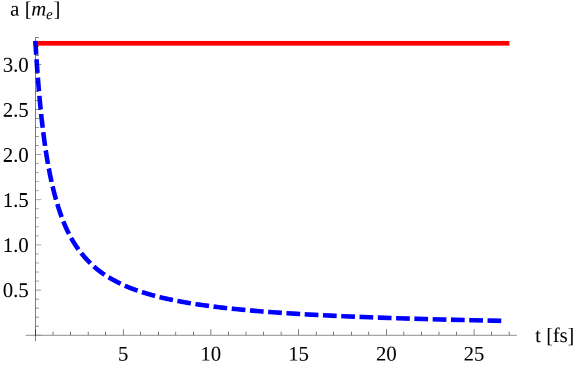
$$m_e\,\dot{u}^\mu = -\frac{e}{c}F^{\mu\nu}u_\nu + F^\mu_{\mathrm{LAD}}$$

$$\begin{aligned} F^\mu_{\mathrm{Caldirola}} &= \frac{m_{ed}}{\tau_a}\left[u^\mu(\tau-\tau_a) - \frac{1}{c^2}\,u^\mu(\tau)u^\alpha(\tau)u_\alpha(\tau-\tau_a)\right] \\ &\approx -m_{\mathrm{ed}}\dot{u}^\mu + \frac{m_{ed}\tau_a}{2}\left[\ddot{u}^\mu + \frac{1}{c^2}\,\dot{u}^\alpha\dot{u}_\alpha u^\mu\right] + \dots \end{aligned}$$

$$m_{\mathrm{ed}} = \frac{2e^2}{3d\epsilon_0 c^2}\,, \quad \tau_a = \frac{2d}{c}$$

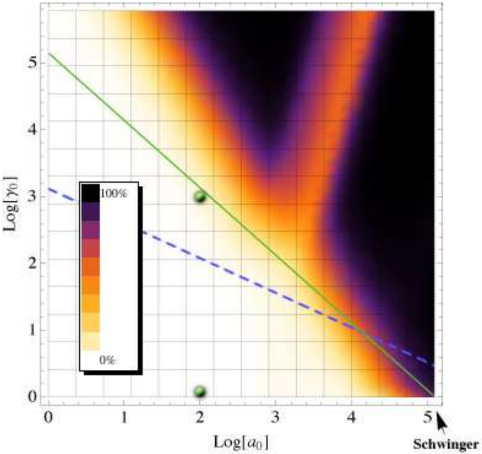
Caldirola used $m_{\mathrm{ed}} \to m_e$. Approach highly useful in PIC (particle in cell) radiation evaluation.

Great difference in dynamics



The Lorentz invariant acceleration $\sqrt{-\dot{u}^\alpha \dot{u}_\alpha}$ arising from solution of dynamical equation as function of laboratory time in natural units for a collision between a circularly polarized laser wave with $a_0 = 100$ and initial $E_e = 0.5\,\mathrm{GeV}, \gamma = 1,000$ electron. The solid red line is the acceleration in the Lorentz force case, while the dashed blue line gives the acceleration according to the Landau-Lifschitz equation.

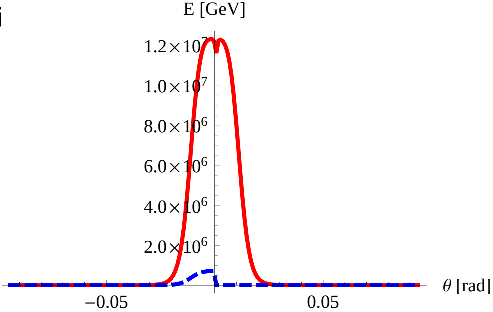
Radiation reaction regime



Deviations from Lorentz force impact significantly Lorentz dynamics in dark shaded area of the γ,a_0 plane

Remark on energy conservation

The RR force is not a conservative force, it acts as a friction force. Even if the force is greater for LL formulation, radiation emissions are stronger for Lorentz force dynamics, mainly because the particle keeps oscillati



Radiation emission for a circularly polarized wave with $a_0 = 100$ colliding head-on with an electron with initial $\gamma_0 = 1,000$. The angle θ is measured on the $x - z$ plane, starting from the negative z-axis. LF equation radiation (solid red line) is one order of magnitude greater than LL equation radiation (the dashed blue line).

Summary

Critical acceleration possible in electron-laser pulse collisions.

Opportunity to study foundational new physics involving acceleration and radiation reaction as dominant force.

Search for generalization of laws of physics – motivated by need for better understanding of inertia, Mach's principle, Einstein's Aether.

Practical: predict greatly amplified and rapid conversion of (electron) beam energy into x-ray and particle pair pulses.