Critical Acceleration

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Sommerfeld Theory Center, LMU, May 25, 2011

In collisions of ultra-intense laser-pulse with relativistic electrons it is possible to experimentally probe critical (Planck) acceleration $a/m = c^3/\hbar$ also achievable when quark-gluon plasma is formed. The behavior of a particle undergoing critical acceleration challenges the limits of the current understanding of basic interactions: survey of electromagnetic radiation reaction, the structure and stability of the quantum vacuum, shows that little is known about that new physics frontier, and that both classical and quantum physics will need further development in order to be able to address this newly accessible area of physics. Connection to strong field particle production, Mach Principle, Unruh and Hawking radiation will be mentioned. Experimental challenges related to the newly available ultra-intense relativistic laser pulse physics will be described. Conference report http://arxiv.org/abs/1010.1970

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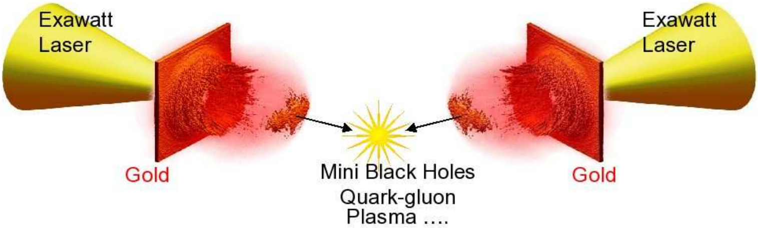
Overview

1 Foundational challenges related to acceleration
2 A novel ‘particle beam’ – the relativistic laser pulse
3 Radiation reaction problem
4 Acceleration of the quantum vacuum state

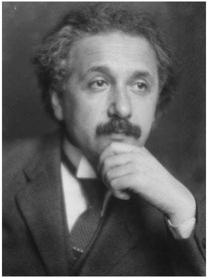


graphics credit to: S.A. Bulanov, G. Mourou, T. Tajima

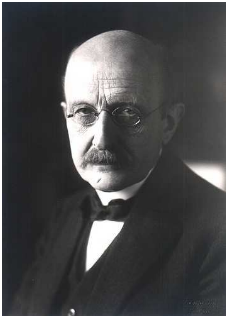
1. Foundational challenges related to acceleration

Critical $\leftrightarrow$ 'Planck' acceleration

Einstein's gravity is built upon Equivalence Principle: a relation of gravity to inertia(=the resistance to acceleration). This suggests that in presence of high acceleration we probe connection between forces (interactions) and the structure of space-time. In that sense there is also a 'limiting' Planck acceleration, which to avoid misunderstanding we term 'critical' acceleration a_c.



1899:
Planck units



$$h/k_B = a = 0.4818 \cdot 10^{-10} [\text{sec} \times \text{Celsiusgrad}]$$
$$h = b = 6.885 \cdot 10^{-27} \left[\frac{\text{cm}^2\text{gr}}{\text{sec}}\right]$$
$$c = c = 3.00 \cdot 10^{10} \left[\frac{\text{cm}}{\text{sec}}\right]$$
$$G = f = 6.685 \cdot 10^{-8} \left[\frac{\text{cm}^3}{\text{gr. sec}^2}\right]^1.$$

Wählt man nun die »natürlichen Einheiten« so, dass in dem neuen Maasssystem jede der vorstehenden vier Constanten den Werth 1 annimmt, so erhält man als Einheit der Länge die Grösse:

$$\sqrt{2\pi}\, L_{Pl} = \sqrt{\frac{bf}{c^3}} = 4.13 \cdot 10^{-33}\,\text{cm}, \mapsto \sqrt{2\pi}\, 1.62 \times 10^{-33}\ \text{cm}$$

als Einheit der Masse:

$$\sqrt{2\pi}\, M_{Pl} = \sqrt{\frac{bc}{f}} = 5.56 \cdot 10^{-5}\,\text{gr}, \mapsto \sqrt{2\pi}\, 2.18 \times 10^{-5}\ \text{g}$$

als Einheit der Zeit:

$$\sqrt{2\pi}\, t_{Pl} = \sqrt{\frac{bf}{c^5}} = 1.38 \cdot 10^{-43}\,\text{sec}, \mapsto \sqrt{2\pi}\, 5.40 \times 10^{-44}\ \text{s}$$

als Einheit der Temperatur:

$$\sqrt{2\pi}\, T_{Pl} = a\sqrt{\frac{c^5}{bf}} = 3.50 \cdot 10^{32}\,{}^\circ\text{Cels.} \mapsto \sqrt{2\pi}\, 1.42 \times 10^{32}\ \text{K}$$

Diese Grössen behalten ihre natürliche Bedeutung so lange bei, als die Gesetze der Gravitation, der Lichtfortpflanzung im Vacuum und die beiden Hauptsätze der Wärmetheorie in Gültigkeit bleiben, sie müssen also, von den verschiedensten Intelligenzen nach den verschiedensten Methoden gemessen, sich immer wieder als die nämlichen ergeben.

“These scales retain their natural meaning as long as the law of gravitation, the velocity of light in vacuum and the central equations of thermodynamics remain valid, and therefore they must always arise, among different intelligences employing different means of measuring.” *M. Planck, “Über irreversible Strahlungsvorgänge.” Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften zu Berlin* **5**, *440-480 (1899), (last page)*

Critical $\rightarrow$ Specific Acceleration

Critical acceleration acting on an electron:

$$a_c = \frac{m_e c^3}{\hbar} \rightarrow 2.331 \times 10^{29} \mathrm{m/s}^2$$

This critical acceleration can be imparted on an electron by the critical 'Schwinger' field strength of magnitude:

$$E_c = \frac{m_e^2 c^3}{e\hbar} = 1.323 \times 10^{18}\, V/m$$

Truly dimensionless unit acceleration arises when we introduce specific acceleration

$$\aleph = \frac{a_c}{mc^2} = \frac{c}{\hbar}$$

Specific unit acceleration arises in Newton gravity at Planck length distance: $\aleph_G \equiv G/L_p^2 = c/\hbar$ at $L_p = \sqrt{\hbar G/c}$. In presence of sufficiently strong electric field E_c by virtue of the equivalence principle, we probe electrons subject to Planck scale force.

Mach and acceleration

- To measure acceleration we need to know an inertial frame of reference.
- Mach's fixed stars are in current context the rest frame of CMB. More generally any cosmologically inertial frame is suitable.

- The full story: Before relativity, Mach posited, as an alternative to Newton's absolute space, a) that inertia (resistance to acceleration) could be due to the background mass in the Universe, and b) that the reference inertial frame must be inertial with respect to the Universe mass rest frame. The latter postulate Einstein called "Mach's principle'.'
- GR and later Cosmology motivated by Machs ideas, yet Einstein nearly eliminated acceleration: GR point masses are in free fall.
- Acceleration returns in presence of quantum matter and non-geometric e.g. quantum forces allowing rigid extended material body. QFT provides the quantum vacuum as a local Mach's frame.

Einstein's Aether as an inertial frame of reference

Albert Einstein at first rejected æther as unobservable when formulating special relativity, but eventually changed his initial position, re-introducing what is referred to as the 'relativistically invariant' æther. In a letter to H.A. Lorentz of November 15, 1919, see page 2 in *Einstein and the Æther*, L. Kostro, Apeiron, Montreal (2000). he writes:

It would have been more correct if I had limited myself, in my earlier publications, to emphasizing only the non-existence of an æther velocity, instead of arguing the total non-existence of the æther, for I can see that with the word æther we say nothing else than that space has to be viewed as a carrier of physical qualities.

In a lecture published in May 1920 (given on 27 October 1920 at Reichs-Universität zu Leiden, addressing H. Lorentz), published in Berlin by Julius Springer, 1920, also in Einstein collected works: In conclusion:

. . . space is endowed with physical qualities; in this sense, therefore, there exists an æther. . . . But this æther may not be thought of as endowed with the quality characteristic of ponderable media, as (NOT) consisting of parts which may be tracked through time. The idea of motion may not be applied to it.

Recollect: Maxwell, Lorentz and inertia are ad-hoc

The action $\mathcal{I}$ comprises three elements: $(F^{\alpha\beta} \equiv \partial^\alpha A^\beta - \partial^\beta A^\alpha)$

$$\mathcal{I} = -\frac{1}{4}\int d^4x\, F^2 + q\int_{\text{path}} d\tau \frac{dx}{d\tau}\cdot A + \frac{mc}{2}\int_{\text{path}} d\tau (u^2 - 1).$$

Path is fixed at the end points assuring gauge-invariance.

• The two first terms upon variation with respect to the field, produce Maxwell equation including radiation emission in presence of accelerated charges.

• The second and third term, when varied with respect to the form of the material particle world line, produce the Lorentz force equation: particle dynamics in presence of fields.

Maxwell and Lorentz equations which arise NEED NOT BE CONSISTENT, the form of $\mathcal{I}$ dictated by gauge- and relativistic-invariance. Many modifications are possible. There is no reference to Mach's inertial frame allowing definition of acceleration.

Gravity and other interactions not truly unified

$$\mathcal{J} = \mathcal{I}_{\sqrt{-g}} + \frac{1}{8\pi G}\int d^4x\sqrt{-g}\,R$$

There are well known acceleration paradoxes arising combining gravity and electromagnetism:

- Charged electron in orbit around the Earth will not radiate if bound by gravitational field, but it will radiate had it been bend into orbit by magnets.
- A free falling electron near BH will not radiate but an electron resting on a surface of a table should (emissions outside observer's horizon.
- A micro-BH will evaporate, but a free falling observer may not see this. Is the BH still there?

One is tempted to conclude that we do not have a theory incorporating acceleration. Opportunity if we can create unit strength (critical) acceleration.

Importance of quantum theory

Without quantum theory we would not have extended bodies, without extended material bodies we cannot create critical acceleration – acceleration due to interplay of quantum and EM theories.

Quantum vacuum state is providing naturally a Machian reference frame lost in classical limit. Critical acceleration in quantum theory (critical fields) leads to new particle production phenomena which have a good interpretation and no classical analog or obvious limit.

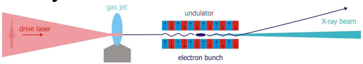
2. EXPERIMENT: A novel ‘particle beam’ – the relativistic laser pulse

New beam in physics: Relativistic laser pulse

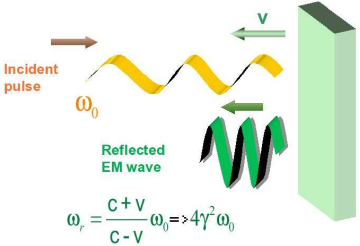
Plasma particle acceleration
Direct particle acceleration

Particle energy grows with pulse intensity squared

Secondaries: e.g.coherent X-rays

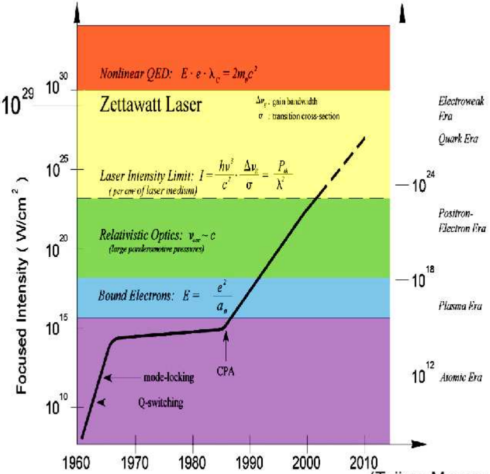


Relativity: coherent γ-ray source



Anything that TeV electrons/positrons can produce colliding with matter: in particular HE neutrinos.

Intensity $I \equiv |\vec{S}|$: 50 years of rise as function of time



(Tajima,Mourou,2002)

What is Intensity:

$$\vec{S} = \frac{c}{4\pi}\vec{E} \times \vec{B}$$

$$I \equiv |\vec{S}| = \frac{\omega k}{4\pi} A_0^2$$

$$\vec{E} = -\partial \vec{A}/c\partial t$$

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$\vec{A} = \mathrm{Re}\{\vec{A}_0 e^{i(\vec{k}\cdot\vec{r} - \omega t)}\}$$

$$A_0 \equiv a_0 mc^2/e$$

Power unit:

$$P_0 = \frac{mc^2}{e}\frac{mc^3}{e} =$$

$$511\mathrm{kV} \cdot 17\mathrm{kA} = 8.67\mathrm{GW}$$

Average pulse intensity:

$$\bar{I} = \pi a_0^2 \bar{P}_0/\lambda^2 =$$

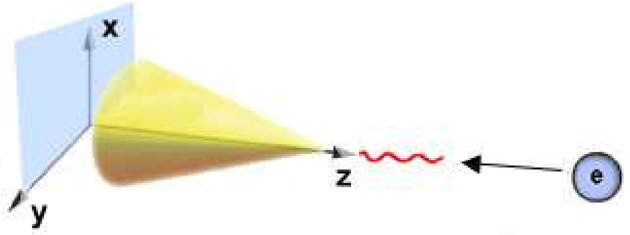
$$2.74\,10^{18}\,\frac{\mathrm{W}}{\mathrm{cm}^2}\,\frac{\mu\mathrm{m}^2}{\lambda^2}\,a_0^2$$

Relativistic regime: $a_0 > 1$

Towards critical (Planck) acceleration

$a = 1 (= m_e c^3/\hbar \to 2.331 \times 10^{29} \mathrm{m/s}^2)$

Directly accelerated electrons at rest in lab by ultra intense laser pulse:
Present day pulse intensity technology misses several orders of magnitude = 10-15y of further development needed. Shortcut: we can Lorentz-boost: to reach the critical acceleration scale today: we collide a counter-propagating electron with a laser pulse.



Laser pulse in electron's rest frame

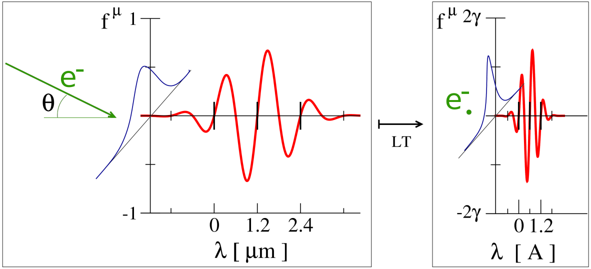


Figure shows boost (from left to right) of the force applied by a Gaussian photon pulse to an electron, on left counter propagating with $\gamma/\cos\theta = 2000$. Pulse narrowed by $(\gamma\cos\theta)^{-1}$ in the longitudinal and $(\gamma\sin\theta)^{-1}$ in the transverse direction. Corresponds to Doppler-shift:

$$\omega \to \omega' = \gamma(\omega + \vec{v}\cdot nk)$$

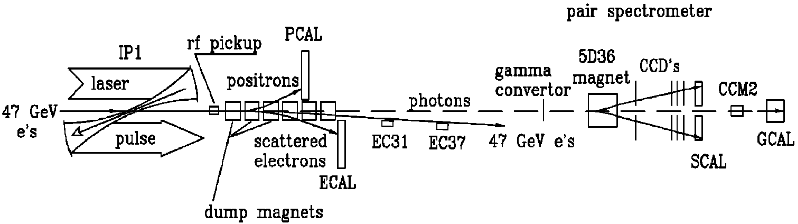
as applied to different frequencies making up the pulse.

Proof of principle: SLAC'95 experiment

$p_e^0 = 46.6$ GeV; in 1996/7 $a_0 = 0.4$, $\left|\frac{du^\alpha}{d\tau}\right| = .073[m_e]$ (Peak)

Multi-photon processes observed:

- Nonlinear Compton scattering
- Breit-Wheeler electron-positron pairs



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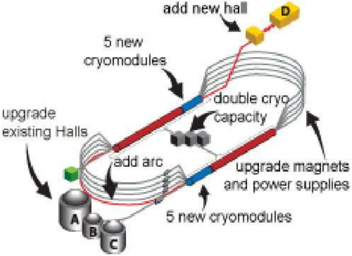
Probing EM-unit acceleration possible today

For example at CEBAF

There is already a 12 GeV ($\gamma = 2400$) electron beam
There is already a laser team
There is appropriate high radiation shielded experimental hall



3. Radiation reaction problem aggravated

Effects of Radiation-Reaction in Relativistic Laser Acceleration;
Y. Hadad, L. Labun, J. Rafelski, (UArizona)
N. Elkina, C. Klier, H. Ruhl, (LMU);
Phys.Rev.D 82, 096012 (2010) arXiv:1005.3980 [hep-ph]

The goal of this paper is twofold:
1) to explore the response of classical charges to electromagnetic force at the level of unity in natural units and
2) to establish a criterion that determines physical parameters for which the related radiation-reaction effects are detectable.

In pursuit of this goal, the Landau-Lifshitz equation is solved analytically for an arbitrary (transverse) electromagnetic pulse. A comparative study of the radiation emission of an electron in a linearly polarized pulse for the Landau-Lifshitz equation and for the Lorentz force equation reveals the radiation-reaction dominated regime, . . .

Acceleration – radiation reaction

We seek a selfconsistent solution of the Maxwell-Lorentz equation system

1) Maxwell Equation: obtain fields $F^{\beta\alpha}$ including radiation fields from a given source of fields j^{α}

$$\partial_\beta F^{*\,\beta\alpha} = 0, \qquad \partial_\beta F^{\beta\alpha} = j^\alpha \quad \to F^{\beta\alpha}$$

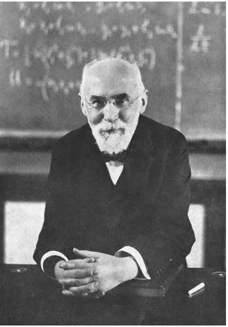
2) Inertial Force = Lorentz-force –> using fields we obtain world line of particles needed in step **1)** (Vlasov: in phase space)

$$m_e \frac{du^\alpha}{d\tau} = -eF^{\alpha\beta}u_\beta \quad \to x^\alpha(\tau),\ \frac{dx^\alpha}{d\tau} \equiv u^\alpha(\tau) \quad \to j^\alpha$$

Solution of Lorentz equation includes now motion in radiation field: that is (selfconsistent) reaction reaction

As long as acceleration is small, radiation emitted can be incorporated as a perturbative additional force. For large acceleration this is a significant new beyond inertia source of resistance to acceleration, there will be additional effective forces besides Lorentz force.

Radiation reaction Lorentz-Abraham-Dirac (LAD) force



energy-momentum radiated:

$$\frac{dp^\alpha}{d\tau} = F^\alpha = -u_\beta e F^{\beta\alpha}_{\mathrm{rad}},$$

$$F^{\beta\alpha}_{\mathrm{rad}} = \frac{2e}{3}(\ddot{u}^\beta u^\alpha - \ddot{u}^\alpha u^\beta)$$

Recognized and further developed among others by

← Lorentz Dirac →

<u>At critical acceleration</u> the radiation impact on charged particle dynamics is a determining factor.

Dirac's 1938 derivation of LAD equation in our language

$$m_e\,\dot{u}^\mu = -\frac{e}{c}F^{\mu\nu}u_\nu\;, \qquad u^\mu = \dot{s}^\mu(\tau)\;, \qquad \partial^\alpha\partial_\alpha A^\mu = \frac{1}{\epsilon_0 c^2}\,j^\mu$$

$$j^\mu(x) = -ec\int d\tau\; u^\mu[s(\tau)]\,\delta^4[x-s(\tau)]$$

$$A^\mu_{\rm rad} = -\frac{e}{\epsilon_0 c}\int d\tau\; u^\mu[s(\tau)]\,G_+[x-s(\tau)]$$

$$-\frac{e}{c}\,F^{\mu\nu}_{\rm rad}\,u_\nu = \frac{e^2}{\epsilon_0 c}\int d\tau\; u_\nu(x)\,\left(u^\nu[s(\tau)]\,\partial^\mu - u^\mu[s(\tau)]\,\partial^\nu\right)\,G_+[x-x(\tau)]$$

$$G_\pm = \theta[\pm X_0]\,\delta[X^2]\;, \qquad X^\mu = x^\mu - x^{\mu\,\prime}$$

$$-\frac{e}{c}\,F^{\mu\nu}_{\rm rad}\,u_\nu = \frac{2e^2}{\epsilon_0 c}\int d\tau\; u_\nu\,\left(u^{\nu\,\prime}X^\mu - u^{\mu\,\prime}X^\nu\right)\,\frac{\partial G_+}{\partial X^2}$$

Expansion for far zone material and EM mass: renormalization

$$X^\mu \approx \delta u^\mu - \frac{\delta^2}{2}\dot{u}^\mu + \frac{\delta^3}{6}\ddot{u}^\mu \pm \ldots , \qquad u^{\mu\,\prime} \approx u^\mu - \delta \dot{u}^\mu + \frac{\delta^2}{2}\ddot{u}^\mu \pm \ldots$$

$$\delta = t - \tau, \qquad X^2 \approx c^2\,\delta^2 \to \frac{\partial}{\partial X^2} = \frac{1}{2c^2\delta}\,\frac{\partial}{\partial \delta}$$

$$F^\mu_{\rm rad} = \frac{e^2}{\epsilon_0 c}\int d\delta\,\frac{\partial G_+}{\partial \delta}\left(\frac{\delta}{2}\dot{u}^\mu - \frac{\delta^2}{3}\left[\ddot{u}^\mu + \frac{1}{c^2}\,\dot{u}^\eta\dot{u}_\eta u^\mu\right]\right)$$

$$= -\frac{e^2}{2\epsilon_0 c^3}\dot{u}^\mu \int d\delta\,\frac{\Delta[\delta]}{|\delta|} + \frac{2e^2}{3\epsilon_0 c^3}\left[\ddot{u}^\mu + \frac{1}{c^2}\,\dot{u}^\eta\dot{u}_\eta u^\mu\right]$$

note $u^2 = 1$ implies $u\cdot\dot{u} = 0$ and thus $\dot{u}^2 = -u\cdot\ddot{u}$

$$m_r\,\dot{u}^\mu = -\frac{e}{c}F^{\mu\nu}_{\rm ext}u_\nu + F^\mu_{\rm LAD}, \qquad m_r = m_e + \frac{e^2}{2\epsilon_0 c^3}\int d\delta\,\frac{\Delta[\delta]}{|\delta|}$$

$$F^\mu_{\rm LAD}u_\mu = 0 \qquad \text{assuring} \qquad u^2 = 1.$$

The causality problem

The appearance of a third derivative $\ddot{u}^\alpha,\ u^\alpha = \dot{x}^\alpha$ requires assumption of an additional boundary condition to arrive at a unique solution describing the motion of a particle.

The usual initial value choice of position and velocity cannot be complemented by choice of initial acceleration, as this will conflict with conventional dynamics in the small acceleration limit. It turns out that a boundary condition in the (infinite) future allows to eliminate accelerating charges, that is ‘run-away’ solutions. Such a constraint is in conflict with the principle of causality.

This LAD radiation reaction description is universally rejected. A theoretical cure is not known, a cottage industry of ad-hoc modifications of radiation-reaction dynamics arose: guess the solution and you will know how to modify theory.

Sample of proposed LAD extensions

LAD	$\mathbf{m\underline{u}}^{\alpha} = \mathbf{qF}^{\alpha\beta}\mathbf{u}_{\beta} + m\tau_0 \left[\ddot{u}^{\alpha} + u^{\beta}\ddot{u}_{\beta}u^{\alpha}\right]$
Landau-Lifshitz	$\mathbf{m\underline{u}}^{\alpha} = \mathbf{qF}^{\alpha\beta}\mathbf{u}_{\beta} + q\tau_0 \left\{F^{\alpha\beta}_{,\gamma}u_{\beta}u^{\gamma} + \frac{q}{m}\left[F^{\alpha\beta}F_{\beta\gamma}u^{\gamma} - (u_{\gamma}F^{\gamma\beta})(F_{\beta\delta}u^{\delta})u^{\alpha}\right]\right\}$
Caldirola	$\mathbf{0} = \mathbf{qF}^{\alpha\beta}(\tau)\,\mathbf{u}_{\beta}(\tau) + \frac{m}{2\tau_0}\left[u^{\alpha}(\tau - 2\tau_0) - u^{\alpha}(\tau)\,u_{\beta}(\tau)\,u^{\beta}(\tau - 2\tau_0)\right]$
Mo-Papas	$\mathbf{m\underline{u}}^{\alpha} = \mathbf{qF}^{\alpha\beta}\mathbf{u}_{\beta} + q\tau_0 \left[F^{\alpha\beta}\dot{u}_{\beta} + F^{\beta\gamma}\dot{u}_{\beta}u_{\gamma}u^{\alpha}\right]$
Eliezer	$\mathbf{m\underline{u}}^{\alpha} = \mathbf{qF}^{\alpha\beta}\mathbf{u}_{\beta} + q\tau_0 \left[F^{\alpha\beta}_{,\gamma}u_{\beta}u^{\gamma} + F^{\alpha\beta}\dot{u}_{\beta} - F^{\beta\gamma}u_{\beta}\dot{u}_{\gamma}u^{\alpha}\right]$
Caldirola-Yaghjian	$\mathbf{m\underline{u}}^{\alpha} = \mathbf{qF}^{\alpha\beta}(\tau)\,\mathbf{u}_{\beta}(\tau) + \frac{m}{\tau_0}\left[u^{\alpha}(\tau - \tau_0) - u^{\alpha}(\tau)\,u_{\beta}(\tau)\,u^{\beta}(\tau - \tau_0)\right]$

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Example: LAD $\rightarrow$ Caldirola

This modification assumes that particle dynamics is non-local: Caldirola notices that LAD derivative terms comprise an effective non locality! This modification creates violation of Lorentz-Poincare symmetry at scales too large to accept, but solves the causality problem. All electron mass $m_{\rm ed} = 2e^2/(3d\epsilon_0 c^2)$ can be in the field. $d \equiv c\tau_a$: a length scale, to be chosen – e.g. so that $m_{\rm ed} = m_{\rm inertial}$. Yagigan variant puts some material mass into electron.

$$
\begin{aligned}
F^\mu_{\rm LAD} \rightarrow F^\mu_{\rm Cald} &= \frac{m_{ed}}{2\tau_a}\left[u^\mu(\tau - 2\tau_a) - \frac{1}{c^2}\,u^\mu(\tau)u^\alpha(\tau)u_\alpha(\tau - 2\tau_a)\right] \\
&\approx -m_{\rm ed}\dot{u}^\mu + m_{ed}\tau_a\left[\ddot{u}^\mu + \frac{1}{c^2}\,\dot{u}^\alpha\dot{u}_\alpha u^\mu\right] + \dots
\end{aligned}
$$

Most popular patch: LAD $\rightarrow$ Landau-Lifshitz (LL)

This modification implies that the field-particle interaction is altered, appropriate action has not been found LL has no conceptual problems and is semi-analytically soluble!

$$m_e \dot{u}^\mu = -\frac{e}{c} F^{\mu\nu} u_\nu + F^\mu_{\rm LAD} \qquad F^\mu_{\rm LAD} = \frac{2e^2}{3\epsilon_0 c^3} \left[\ddot{u}^\mu + \frac{1}{c^2} \dot{u}^\eta \dot{u}_\eta u^\mu \right]$$

Iterate using Lorentz force and its differential in LAD:

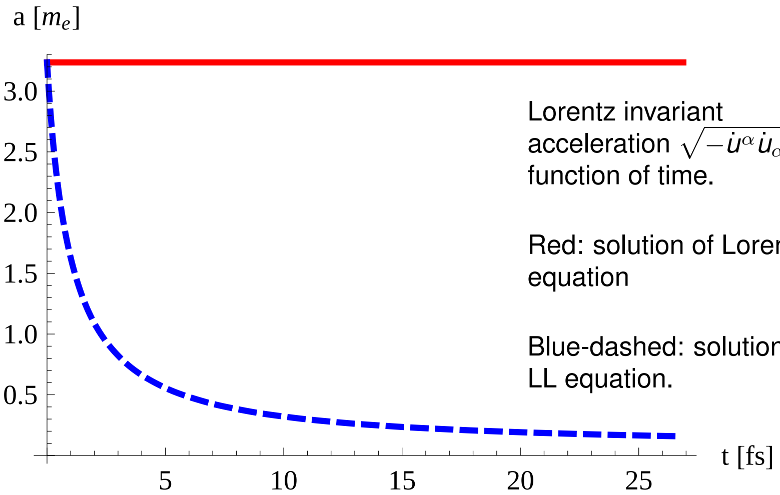
$$\begin{aligned} \ddot{u}^\mu &\rightarrow \frac{d}{d\tau}\left(-\frac{e}{m_e c} F^{\mu\nu} u_\nu \right) = -\frac{e}{m_e c} \left(\partial_\eta F^{\mu\nu} u_\nu u^\eta + F^{\mu\nu} \dot{u}_\nu \right) \\ &= -\frac{e}{m_e c} \left(\partial_\eta F^{\mu\nu} u_\nu u^\eta - \frac{e}{m_e c} F^{\mu\nu} F^\eta_\nu u_\eta \right) \end{aligned}$$

$$F^\mu_{\rm LAD} \simeq -\frac{2e^3}{3\epsilon_0 m_e c^4} \left(\partial_\eta F^{\mu\nu} u_\nu u^\eta - \frac{e}{m_e c} F^{\mu\nu} F^\eta_\nu u_\eta \right) + \frac{2e^4}{3\epsilon_0 m_e^2 c^7} F^{\eta\nu} F_{\eta\delta} u_\nu u^\delta u^\mu$$

This is equivalent to LAD only for weak acceleration.

Example: acceleration in pulse-e collisions

Collision between a circularly polarized laser wave with $a_0 = 100$ and initial $E_e = 0.5\,\mathrm{GeV}, \gamma = 1,000$ electron.

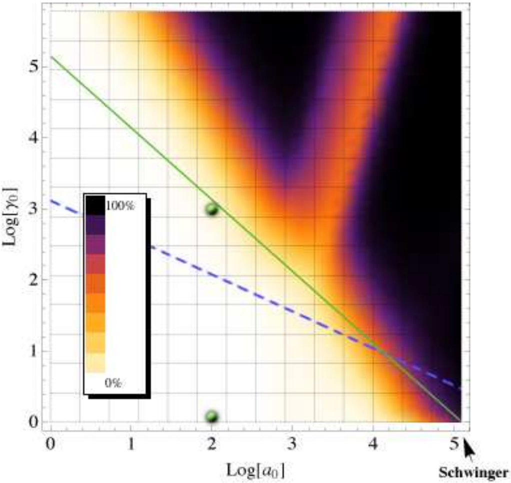


Lorentz invariant acceleration $\sqrt{-\dot{u}^{\alpha}\dot{u}_{\alpha}}$ as function of time.

Red: solution of Lorentz equation

Blue-dashed: solution of LL equation.

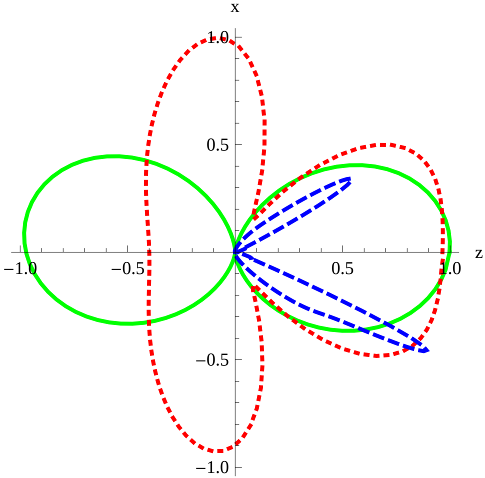
Radiation reaction (RR) regime



Dark shaded area in $\gamma > 1, a_0 > 1$ plane, : dynamics dominated by radiation reaction force, according to solution of the LL version of radiation reaction. For $a_0 \to 0$ this occurs at critical Schwinger field $E_c = m^2/e$

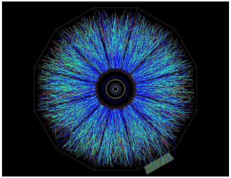
Radiation intensity patterns

This plot is identical for the LF and for the LL Equation.



The angular distribution of radiation for a linearly polarized wave. This is the normalized radiation distribution for an electron initially at rest, after interacting with a laser with $a_0 = 0.1$, $a_0 = 1$ and $a_0 = 10$ plotted in solid green, dotted red and dashed blue lines respectively.

Critical acceleration in hadron collisions



Two nuclei smashed into each other from two sides at highest achievable energy: components can be stopped in CM frame within $\Delta\tau \simeq 1$ fm/c. Tracks show multitude of particles produced, as observed by STAR at RHIC (BNL).

- The acceleration *a* achieved to stop some/any of the components of the colliding nuclei in CM: $a \simeq \frac{\Delta y}{M_i \Delta\tau}$. Full stopping: $\Delta y_{\rm SPS} = 2.9$, and $\Delta y_{\rm RHIC} = 5.4$. Considering constituent quark masses $M_i \simeq M_N/3 \simeq 310$ MeV we need $\Delta\tau_{\rm SPS} < 1.8$ fm/c and $\Delta\tau_{\rm RHIC} < 3.4$ fm/c to exceed a_c.
- Observed unexplained soft electromagnetic radiation in hadron reactions *A. Belognni et al. [WA91 Collaboration], "Confirmation of a soft photon signal in excess of QED expectations in* π*–p interactions at 280-GeV/c," Phys. Lett. B* **408**, *487 (1997)*

. • Recent suggestions that thermal hadron radiation due to Unruh type phenomena *P. Castorina, D. Kharzeev and H. Satz, "Thermal Hadronization and Hawking-Unruh Radiation in QCD," Eur. Phys. J. C* **52**, *187 (2007)*

4. Acceleration and the quantum vacuum state

Acceleration and temperature

- Unruh observes that an accelerated observers will perceive temperature

  $$T_U = \frac{a}{2\pi}$$

  relation to Hawking radiation:
  strong acceleration $=$?? strong gravity
- Properties of quantum vacuum ‘accelerated’ in a constant field can be cast into a form displaying heat capacity at temperature

  $$T_E = \frac{a}{\pi}, \qquad a = eE/m$$

  resolution of factor 2 and of the related issue of Fermi-Bose statistics choice remain a challenge

Euler-Heisenberg Z. Physik 98, 714 (1936) evaluate Dirac zero-point energy with (constant on scale of $\hbar/mc$) EM- fields, transform to action
$\mathcal{E}(D,H) \to \mathcal{L}(E,B) = \mathcal{E} - ED$, $E \equiv \frac{\partial \mathcal{E}}{\partial D}$

$$\mathcal{L}(E,B) = -\frac{1}{8\pi^2}\int_{i\epsilon}^{\infty} \frac{ds}{s^3}\mathrm{e}^{-m^2 s}\left[\frac{sE}{\tan sE}\,\frac{sB}{\tanh sB} - 1 + \frac{1}{3}(E^2 - B^2)s^2\right].$$

E, B relativistic definition in terms of invariants

$$\mathcal{L}(E,B) \to \frac{2\alpha^2}{45m^4}\left[\left(\vec{E}^{\,2} - \vec{B}^{\,2}\right)^2 + 7(\vec{E}\cdot\vec{B})^2\right] = \mathrm{light} - \mathrm{light\ scattering}$$

Schwinger 1951 studies in depth the imaginary part – see zeros in $\tan sE$:

$$|\langle 0_+|0_-\rangle|^2 = e^{-2\mathcal{I}m\,\mathcal{L}}, \qquad 2\mathcal{I}m\,\mathcal{L} = \frac{\alpha^2}{\pi^2}\sum_{n=1}^{\infty} n^{-2} e^{-n\pi m^2/eE}$$

is the vacuum persistence probability in adiabatic switching on/off the E-field. For $E \to 0$ essential singularity.

Note: vacuum unstable for any finite value of E and there is no analytic $E \to 0$ limit!. The light–light perturbative term is first term of semi convergent expansion which fails in a subtle way.

Temperature for vacuum fluctuations at a =Const.

Using identity [B.Müller, W.Greiner and JR, PLB63A (1977) 181]:

$$\frac{x\cos x}{\sin x} = 1 - \frac{x^2}{3} + \sum_{k=1} \frac{1}{k^2\pi^2}\frac{2x^4}{x^2 - k^2\pi^2}$$

give after resummation a "statistical" form with $T = eE/\pi m = a/\pi$:

$$\mathcal{L}(E) = -2_s \frac{m^2 T}{8\pi^2}\int_0^\infty \rho(\omega)\ln(1 - e^{-\omega/T})d\omega, \quad \rho = \ln(\omega^2 - m^2 + i\epsilon);$$

Same sign of pole residues source of Bose statistics. For scalar loop particles we have $\frac{x}{\sin x} = 1 + \frac{x^2}{6} + \sum_{k=1} \frac{(-1)^k}{k^2\pi^2}\frac{2x^4}{x^2 - k^2\pi^2}$ which yields same expression up to: 1) statistics turns from Bose into Fermi: $-\ln(1 - e^{-\omega/T}) \to \ln(1 + e^{-\omega/T})$ and 2) factor $2_s \to 1$. Note that the statistics is reversed compared to expectations and temperature is twice larger compared to Hawking-Unruh observer. Potential improvements needed in the nonperturbative approach to constrain symmetry of the vacuum state.

Summary

New opportunity to study foundational physics involving acceleration and search for generalization of laws of physics – motivated by need for better understanding of inertia, Mach's principle, Einstein's Aether, and the temperature of the accelerated quantum vacuum .

Critical acceleration possible in electron-laser pulse collisions.
Exploration of physical laws in a new domain possible

Experiments should help resolution the old riddle of EM theory and radiation reaction
Rich field of applications follows

Better understanding of particle collisions: radiation reaction may be origin of jet quenching.