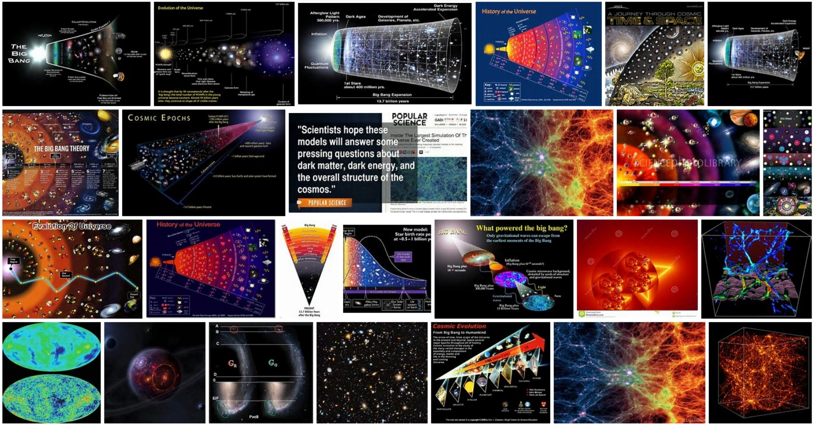
New Insights into the Time Evolution of the Universe

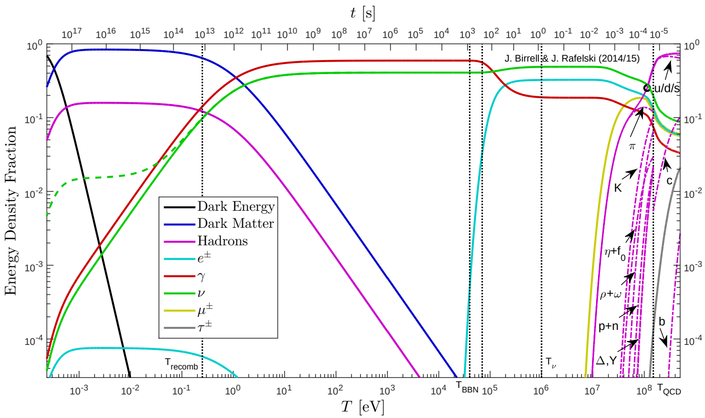
John W Clark, Wayman Crow Professor of Physics



Stunn

graphics! But the key result missing:

The time evolution of the energy density composition


t [s]
J. Birrell & J. Rafelski (2014/15)
Energy Density Fraction
T [eV]
Dark Energy
Dark Matter
Hadrons
e±
γ
ν
μ±
τ±
T recomb
T BBN
T ν
T QCD
u/d/s
π
K
c
η+f0
ρ+ω
p+n
Δ,Y
b

Input into the image

- FRW Cosmology
- Disappearing Particles:
  Degrees of Freedom and Reheating – tracking T_γ
- Connecting the Eras
  - From the beginning to QGP – remarks
  - And matter free-streams, latest when:
  - QGP turns into disappearing hadrons, invisible radiation,. . .
  - Onset of neutrino free-streaming
  - Big-Bang nucleosynthesis and disappearance of matter
  - Emergence of free streaming dark matter, baryons follow
  - Photon Freestreaming – Composition Cross-Point
  - Dark Energy Emerges – vacuum energy
- Open questions abound

Overview: the Friedmann–Lemaitre–Robertson–Walker (FRW) cosmology assumes

- Homogeneous
- Isotropic

Einstein Universe

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2(\theta)d\phi^2)\right].$$

Flat ($k=0$) metric is favored [1] in the ΛCDM analysis. $a(t)$ determines the distance between objects at rest in the Universe frame (comoving). Skipping $g^{\mu\nu} \to R^{\mu\nu}$

[1] *Planck Collaboration, Astron. Astrophys.* **571**, *A16 (2014) [arXiv:1303.5076] and arXiv:1502.01589 [astro-ph.CO].*

Einstein's Equations, where $R = g_{\mu\nu}R^{\mu\nu}$:

$$G^{\mu\nu} = R^{\mu\nu} - \left(\frac{R}{2} + \Lambda\right) g^{\mu\nu} = 8\pi G_N T^{\mu\nu}, \quad T^{\mu}_{\nu} = \mathrm{diag}(\rho, -P, -P, -P),$$

We absorb the vacuum energy (Einstein Λ-term) into the energy ρ and pressure P

$$\rho \to \rho + \rho_\Lambda, \qquad P \to P + P_\Lambda$$

which contain other components in the Universe including CDM: cold dark matter. The ΛCDM speaks thus of Dark Energy, or equivalently, non-vanishing vacuum energy density

$$\rho_\Lambda \equiv \Lambda/(8\pi G_N) = 25.6\ \mathrm{meV}^4, \qquad P_\Lambda = -\rho_\Lambda$$

The pressure P_Λ has a) opposite sign from all matter contributions and b) $\rho_\Lambda/P_\Lambda = -1$.

Definitions: Hubble parameter H and deceleration parameter q:

$$H(t) \equiv \frac{\dot{a}}{a}; \quad q \equiv -\frac{a\ddot{a}}{\dot{a}^2} = -\frac{1}{H^2}\frac{\ddot{a}}{a}, \Rightarrow \dot{H} = -H^2(1+q).$$

Two dynamically independent Einstein equations arise

$$\frac{8\pi G_N}{3}\rho = \frac{\dot{a}^2 + k}{a^2} = H^2\left(1 + \frac{k}{\dot{a}^2}\right), \quad \frac{4\pi G_N}{3}(\rho + 3P) = -\frac{\ddot{a}}{a} = qH^2.$$

solving both these equations for $8\pi G_N/3 \rightarrow$ we find for the deceleration parameter:

$$q = \frac{1}{2}\left(1 + 3\frac{P}{\rho}\right)\left(1 + \frac{k}{\dot{a}^2}\right).$$

In flat $k = 0$ Universe: ρ fixes H; with P also q fixed, and thus also $\dot{H}$ fixed so also $\dot{\rho}$ fixed, and therefore also for $\rho = \rho(T(t))$ also $\dot{T}$ fixed.

The contents of the Universe:

- Matter coupled to photons: thermal matter = ideal Bose-Fermi gases
- Free-streaming matter: today
  - dark matter: since at or before QGP hadronization
  - neutrinos: since $T =$few MeV
  - photons: since $T = 0.25eV$
  - hypothetical darkness: quasi-massless particles, like neutrinos but due to earlier decoupling small impact on Universe dynamics
- dark energy = vacuum energy

Degrees of Freedom

The effective number of entropy degrees of freedom, g_*^S, defined by

$$S = \frac{2\pi^2}{45} g_*^S T_\gamma^3 a^3.$$

For ideal Fermi and Bose gases

$$g_*^S = \sum_{i=\text{bosons}} g_i \left(\frac{T_i}{T_\gamma}\right)^3 f_i^- + \frac{7}{8} \sum_{i=\text{fermions}} g_i \left(\frac{T_i}{T_\gamma}\right)^3 f_i^+.$$

g_i are the degeneracies, $f_i^\pm$ are functions varying valued between 0 and 1 that turn off the various species as the temperature drops below their mass.

Degrees of Freedom

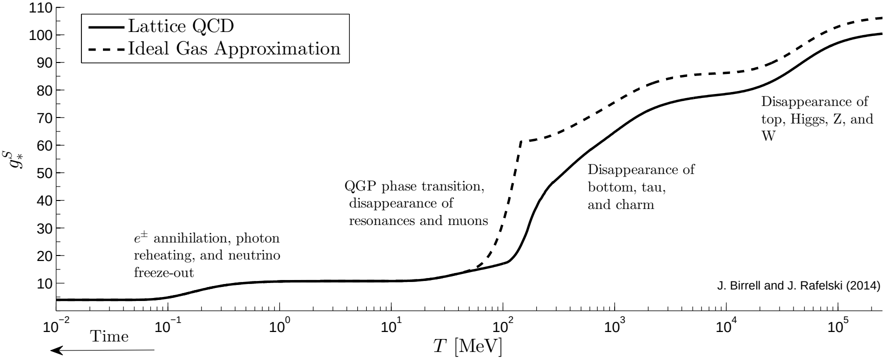


Figure: Ideal gas approximation is not valid during QGP phase transition and equation of state from lattice QCD must be used [1]. At and above 300 MeV non-rigorous matching [2] with perturbation calculations may impact result.

[1] *S. Borsanyi, Nucl. Phys. A904-905, 270c (2013)*

[2] *Mike Strickland (private communication of results and review of thermal SM).*

Conservation of Entropy and Reheating

Once (example) Darkness decouples from SM particles at a photon temperature of $T_{d,s}$, a difference in its temperature from that of photons will build up during subsequent reheating periods. Conservation of entropy leads to a temperature ratio at $T_\gamma < T_{d,s}$ of

$$R_s \equiv T_s/T_\gamma = \left(\frac{g_*^S(T_\gamma)}{g_*^S(T_{d,s})}\right)^{1/3}.$$

This can be used to determine the present day reheating ratio as a function of decoupling temperature throughout the Universe history.

Reheating and Particle Disappearance History

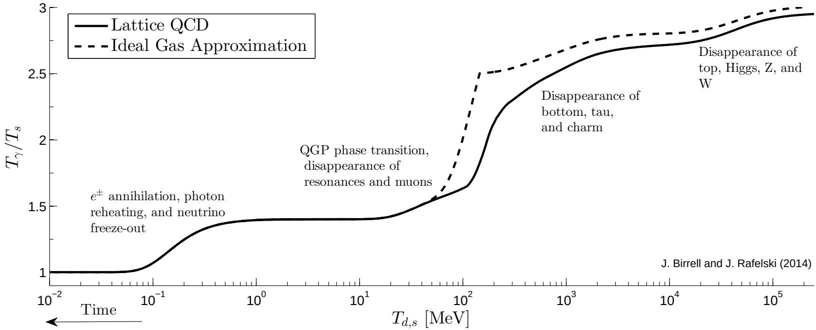


Figure: Cube of the present day ratio of photon temperature to the temperature of a particle species that decoupled at $T_{d,s}$. The reheating ratio reflects the disappearance of degrees of freedom from the Universe. At and above 300 MeV non-rigorous matching with perturbation calculations may impact result.

Free-streaming matter: solution of kinetic equations with decoupling boundary conditions at T_k (kinetic freeze-out)

$$\rho = \frac{g_\nu}{2\pi^2}\int_0^\infty \frac{\left(m_\nu^2+p^2\right)^{1/2} p^2 dp}{\Upsilon_\nu^{-1} e^{\sqrt{p^2/T_\nu^2+m_\nu^2/T_k^2}}+1}, \quad P = \frac{g_\nu}{6\pi^2}\int_0^\infty \frac{\left(m_\nu^2+p^2\right)^{-1/2} p^4 dp}{\Upsilon_\nu^{-1} e^{\sqrt{p^2/T_\nu^2+m_\nu^2/T_k^2}}+1},$$

$$n = \frac{g_\nu}{2\pi^2}\int_0^\infty \frac{p^2 dp}{\Upsilon_\nu^{-1} e^{\sqrt{p^2/T_\nu^2+m_\nu^2/T_k^2}}+1}.$$

These differ from the corresponding expressions for an equilibrium distribution by the replacement $m \to mT_\nu(t)/T_k$ *only* in the exponential. Only for massless photons free-streaming = thermal distributions.

C. Cercignani, and G. Kremer. The Relativistic Boltzmann Equation: Basel, (2000).
H. Andreasson, "The Einstein-Vlasov System"Living Rev. Rel. **14**, *4 (2011) Y. Choquet-Bruhat. General Relativity and the Einstein Equations, Oxford (2009).*

Identifying Eras by Deceleration Parameter

- Radiation dominated universe: $P = \rho/3 \implies q = 1$.
- Matter dominated universe: $P \ll \rho \implies q = 1/2$.
- Dark energy (Λ) dominated universe: $P = -\rho \implies q = -1$.

Hadron and QGP Era

- QGP era down to phase transition at $T \approx 150$MeV. Energy density dominated by photons, neutrinos, $e^{\pm}$, $\mu^{\pm}$ along with u,d,s.
- $2+1$-flavor lattice QCD equation of state must be used $[1]$.
- u,d,s lattice energy density is matched by ideal gas of hadrons to sub percent-level at $T = 115$MeV.
- Hadrons included: pions, kaons, eta, rho, omega, nucleons, delta, Y
- Hadron pressure is discontinuous at 10% level. Causes hard to notice discontinuity in q (slops match).

[1] *S. Borsanyi, Nucl. Phys. A904-905, 270c (2013)*

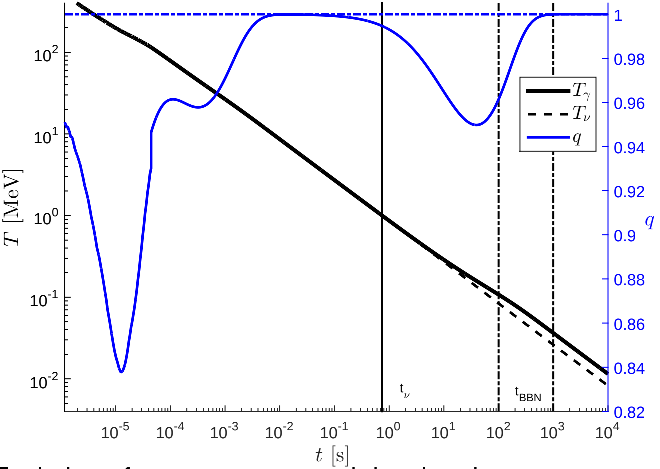
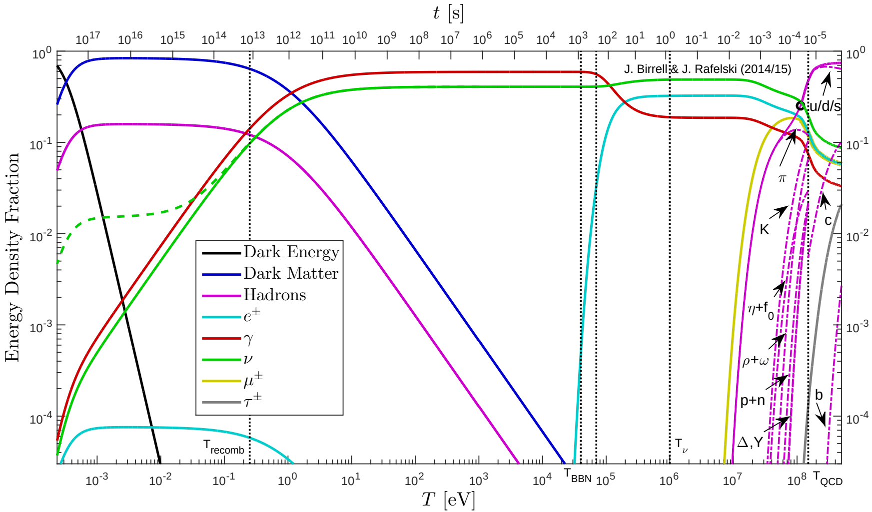


Figure: Evolution of temperature T and deceleration parameter q from QGP era until near BBN.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Dark Energy
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Hadrons
$e^{\pm}$
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T_{BBN}
T_{ν}
T_{QCD}
u/d/s
π
K
c
η+f_0
$\rho+\omega$
p+n
Δ,Y
b

Pioneering work done 43 years ago:

VOLUME 29, NUMBER 10 PHYSICAL REVIEW LETTERS 4 SEPTEMBER 1972

An Upper Limit on the Neutrino Rest Mass*

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Department of Physics, University of California, Berkeley, California 94720
(Received 17 July 1972)

In order that the effect of graviation of the thermal background neutrinos on the expansion of the universe not be too severe, their mass should be less than 8 eV/c^2.

Is Understanding of Neutrino Freeze-out Accurate?

- The computed best value is $N_\nu = 3.046$ (some flow of $e^{\pm}$-pair into ν) [1]. Only drastic changes in neutrino properties and/or physical laws can change this value noticeably [2].
- δN_ν also probes 'Darkness' particle content in the Universe: new relativistic particles in the early Universe modify N_ν, see e.g. [3].
- δN_ν limits variation of fundamental constants in early Universe [4]

[1] *G. Mangano et. al., Nucl. Phys. B* **729**, *221 (2005)*
[2] *J. Birrell, C. T. Yang and JR, Nucl. Phys. B* **890**, *481 (2014) [1406.1759 [nucl-th]]*
[3] *Steven Weinberg Phys. Rev. Lett.* **110**, *241301 (2013)*
[4] *J. Birrell, C-T Yang, J. Rafelski Nucl.Phys. B* **890** *481-517 (2014)*

Relic Neutrino Background:

At a temperature of 5 MeV the Universe consisted of $e^{\pm}$-pairs, photons, and neutrino plasma. At around 1 MeV neutrinos stop interacting or freeze-out and free stream through the universe. Today they comprise the relic neutrino background (CNB).

Direct measurement:
Relic neutrinos have not been directly measured.

Indirect measurement:
Impact on speed of Universe expansion can be seen in the CMB. This constrains neutrino mass and number of invisible relativistic degrees of freedom dominated by cosmic neutrinos.

Radiation Dominated Era

- Neutrinos freeze-out at $T \approx 1$MeV.
- A small deviation from radiation dominated during $e^{\pm}$ annihilation.
- Energy density dominated by neutrinos, photons down through BBN ($T = 40 - 70$keV) until $T = O(1\text{eV})$

Dark energy and Matter Dominated Eras

- Present day on left of plot: 69% dark energy, 26% dark matter, 5% baryons, $< 1\%$ photons and neutrinos.
- Solid neutrino line shows massless neutrinos. Dashed line shows 1 massless and 2×0.1 eV neutrinos (Neutrino mass choice is just for illustration. Other values are possible)
- First vertical line on the left shows recombination at $T \approx 0.25$eV.

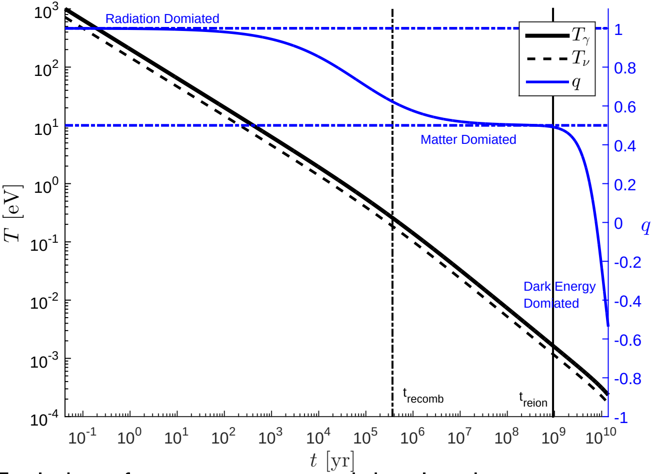

Figure: Evolution of temperature T and deceleration parameter q from soon after BBN to the present day

Work in progress: Dark Matter Decoupling

- IF DM freeze-out such that $M \lesssim T_f$ (neutrino-like) then DM would constitute a large fraction of the energy density during BBN and spoil the results. In this case
- We need h that DM disappears before BBN, such DM would be irrelevant in present day energy balance.
- Therefore we think BBN implies condition $T_f \ll M$ (electron-positron like) with DM either stable or slowly reduced by decay, annihilation, etc , with a persistence time on the order of the age of the Universe or longer.
- Such $T_f \ll M$ freeze-out condition requires either visible interaction (forbidden by 'D'), or it is driven by vacuum transport property a.k.a quark confinement.

Dark Matter free-streaming equation of State

The energy density of a free-streaming species of fermionic DM with degeneracy g_{DM} and mass M that froze out at a temperature T_f is [1]

$$\rho_{DM} = \frac{g_{DM}}{2\pi^2}\int_0^\infty \frac{\left(M^2+p^2\right)^{1/2} p^2 dp}{\Upsilon^{-1} e^{\sqrt{p^2/T_{DM}^2+M^2/T_f^2}}+1},$$

$$P_{DM} = \frac{g_{DM}}{6\pi^2}\int_0^\infty \frac{\left(M^2+p^2\right)^{-1/2} p^4 dp}{\Upsilon^{-1} e^{\sqrt{p^2/T_{DM}^2+M^2/T_f^2}}+1},$$

$$n_{DM} = \frac{g_{DM}}{2\pi^2}\int_0^\infty \frac{p^2 dp}{\Upsilon^{-1} e^{\sqrt{p^2/T_{DM}^2+M^2/T_f^2}}+1}.$$

[1] *J. Birrell, C. T. Yang, P. Chen and J. Rafelski, Phys. Rev. D* **89** *(2014)*

Free-Streaming Dark Matter: Non-Relativistic Limit

Let $E(p) = p^2/2M$, $T_\gamma/T_{DM} = R_{DM}^{1/3}$.

$$n_{DM} \approx g_{DM}\, T_{DM}^3 e^{-M/T_f} \left(\frac{M}{2\pi T_f}\right)^{3/2},$$

$$\rho_{DM} \approx M\, n_{,DM}$$

$$P_{DM} \approx \frac{T_{DM}^2}{T_f}\, n_{DM}.$$

Dark Matter: Non-Relativistic Limit

Using the DM abundance and flatness ($k = 0$) results from Planck we compute the present day dark matter density

$$\rho_{DM} = .26\rho_{\text{crit}} = 9.64 \text{ meV}^4 = M\, n_{DM}$$

This yields a relation between T_f and M,

$$g_{DM}M = \frac{(2\pi)^{3/2}\rho_{DM}}{T_\gamma^3} r_U(T_f) e^{M/T_f} \left(\frac{T_f}{M}\right)^{3/2}.$$

The density of dark matter dilutes with universe expansion, and this is accounted for above taking the energy density from freeze-out to present day condition. Ratio of Dark to photon temperature introduces reheating factor which carries implicit dependence on T_f

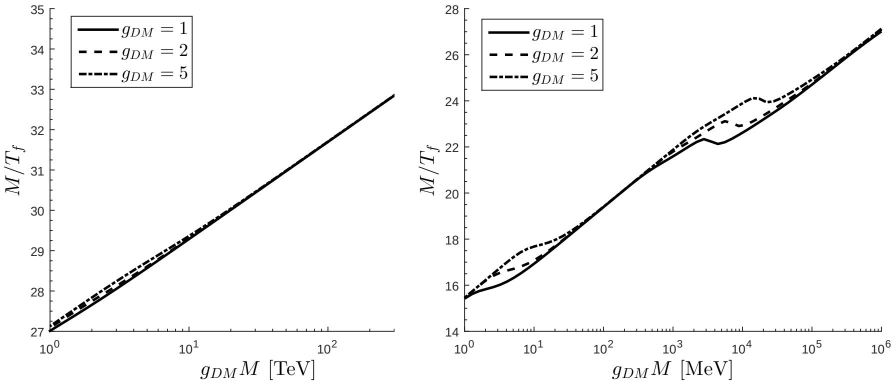

Figure: Combinations of dark matter mass and freeze-out temperatures that yield the present day CDM energy density fraction of 26%.

J. Birrell, et al "Relic Neutrino Freeze-out: Dependence on Natural Constants," Nucl. Phys. B **890** (2014) 481 [arXiv:1406.1759 [nucl-th]].

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J. Birrell, J. Wilkening and J. Rafelski, "Boltzmann Equation Solver Adapted to Emergent Chemical Non-equilibrium," J. Comput. Phys. **281** (2014) 896 [arXiv:1403.2019 [math.NA]].

J. Birrell and J. Rafelski, "Proposal for Resonant Detection of Relic Massive Neutrinos," Eur. Phys. J. C **75** (2015) 2, 91 [arXiv:1402.3409 [hep-ph]].

J. Rafelski and J. Birrell, "Traveling Through the Universe: Back in Time to the Quark-Gluon Plasma Era," J. Phys. Conf. Ser. **509** (2014) 012014 [arXiv:1311.0075 [nucl-th]].

J. Birrell, et al Mod. Phys. Lett. A **28** (2013) 1350188 [arXiv:1303.2583 [astro-ph.CO]].

J. Birrell, et al Phys. Rev. D **89** (January 2014) 023008 [arXiv:1212.6943 [astro-ph.CO]].

J. Rafelski, L. Labun and J. Birrell, "Compact Ultradense Matter Impactors," Phys. Rev. Lett. **110** (2013) 11, 111102 [arXiv:1104.4572 [astro-ph.EP]].

Stunn graphics! But the key result missing:

40 years of friendship

Geneva Lake shore after colloquium 1978