Special Relativity Matters:
Magnetic moment dynamics

Introduction

Teaching Special Relativity

Body Contraction

Forces and acceleration

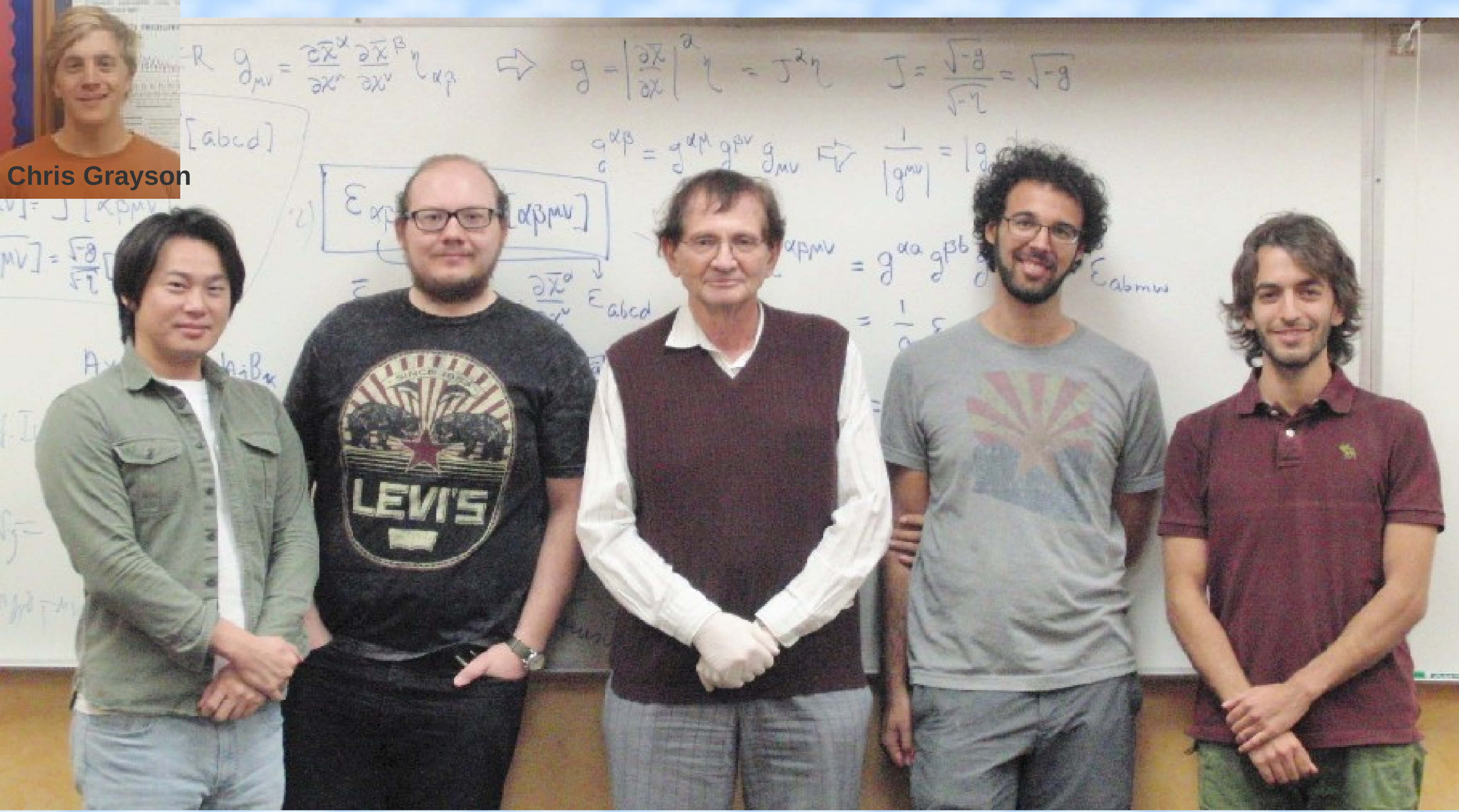
Mach's principle

Aether=Quantum Structured Vacuum

Acceleration Frontier

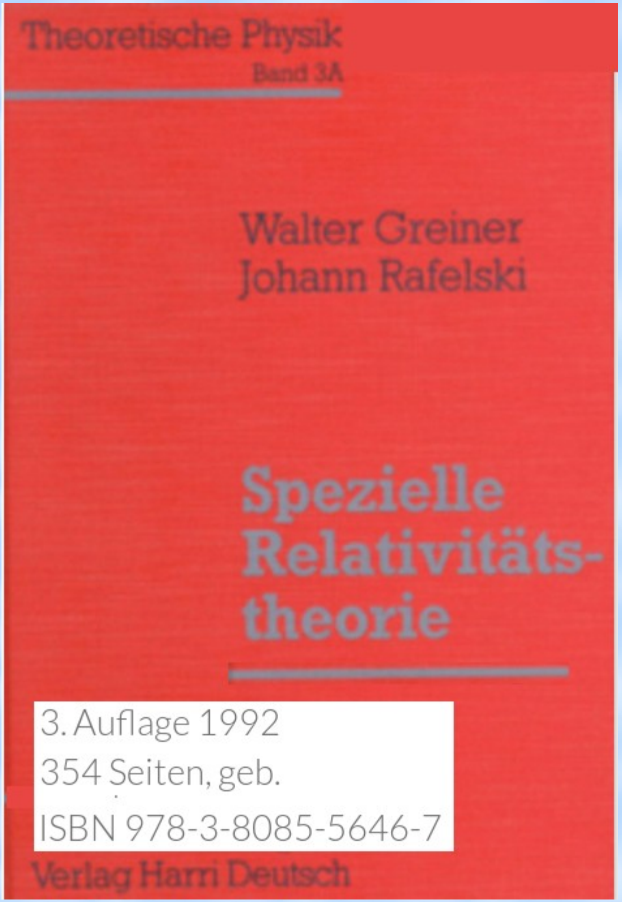
Radiation-Reaction

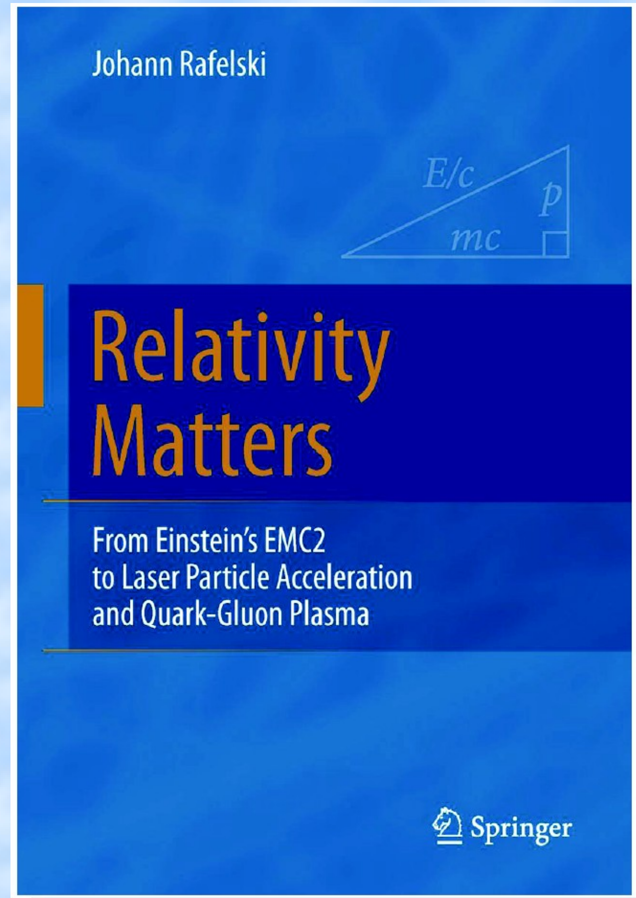
Stern-Gerlach magnetic moment force



Cheng Tao Young, Andrew Steinmetz, Jan R, Martin Formanek, Stefan Evans

Relativity Matters:
Long Interest in teaching SR





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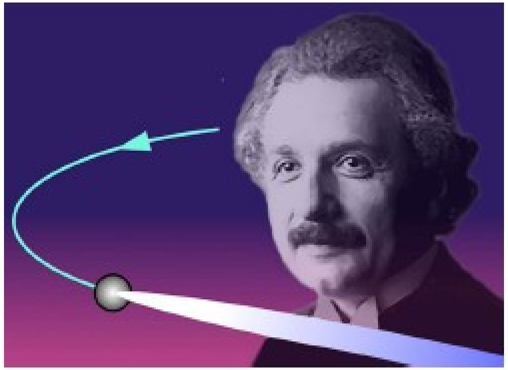
Keywords: special relativity; Doppler; time dilation; Lorentz transformation.



OPEN ACCESS

The relativistic foundations of synchrotron radiation

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Special relativity (SR) determines the properties of synchrotron radiation, but the corresponding mechanisms are frequently misunderstood. Time dilation is often invoked among the causes, whereas its role would violate the principles of SR. Here it is shown that the correct explanation of the synchrotron radiation properties is provided by a combination of the Doppler shift, not dependent on time dilation effects, contrary to a common belief, and of the Lorentz transformation into the particle reference frame of the electromagnetic field of the emission-inducing device, also with no contribution from time dilation. Concluding, the reader is reminded that much, if not all, of our argument has been available since the inception of SR, a research discipline of its own standing.

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THE EUROPEAN
PHYSICAL JOURNAL A

Letter

Measurement of the Lorentz-FitzGerald body contraction*

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Received: 25 June 2017 / Revised: 16 August 2017
Published online: 20 February 2018

Communicated by T. Biro

Dedicated to Walter Greiner; October 1935 – October 2016.

Abstract. A complete foundational discussion of acceleration in the context of Special Relativity (SR) is presented. Acceleration allows the measurement of a Lorentz-FitzGerald body contraction created. It is argued that in the back scattering of a probing laser beam from a relativistic flying electron cloud mirror generated by an ultra-intense laser pulse, a first measurement of a Lorentz-FitzGerald body contraction is feasible.

Introduction

Within the relativity framework created by Einstein in 1905 [1], there is no acceleration. Einstein considered only inertially moving bodies and observers. Imposing Galileo's relativity principle, homogeneity and isotropy of space, and the constancy of the speed of light, Einstein obtained all his results using Lorentz coordinate transformations that follow from these principles. To obtain the relativistic Doppler effect maintaining the relativity principle, Einstein postulated that the light wave phase is a Lorentz in-

This situation is described in 1960 by Rindler [3]: "Relativity offers no detailed explanation in terms of cohesive forces or the like (*however, compare* [4], *Chapt. 10 Discussion IV-2*), yet it predicts the contraction phenomenon as inevitable. This is comparable to some of the predictions based on the energy principle. It must be stressed that the phenomenon is not to be regarded as illusory ... it is real in every possible sense." The last comments echo the remarks of Einstein of 1911 [2]. Lajos Jánossy in his 1971 book calls Lorentz approach "physical reality" [5].

In 1976 John S. Bell, of quantum Bell-inequality fame,

Eur. Phys. J. C (2018) 78:6
https://doi.org/10.1140/epjc/s10052-017-5493-2

THE EUROPEAN PHYSICAL JOURNAL C

CrossMark

Regular Article - Theoretical Physics

Relativistic dynamics of point magnetic moment

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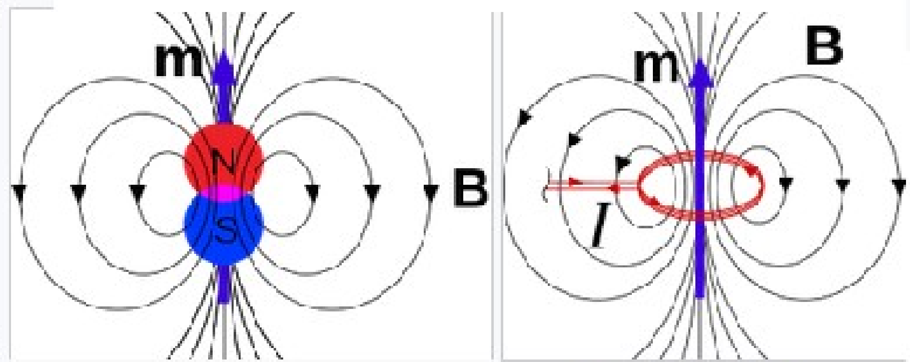


Dipole moment **m** Electric current *I*

The **magnetic field** and **magnetic moment**, due to natural magnetic dipoles (left), or an electric current (right). Either generates the same field profile.

Received: 1 December 2017 / Accepted: 19 December 2017

Abstract The covariant motion of a classical point particle with magnetic moment in the presence of (external) electromagnetic fields is revisited. We are interested in understanding extensions to the Lorentz force involving point particle magnetic moment (Stern–Gerlach force) and how the spin precession dynamics is modified for consistency. We introduce spin as a classical particle property inherent to Poincaré symmetry of space-time. We propose a covariant formulation of the magnetic force based on a 'magnetic' 4-potential and show how the point particle magnetic moment

1. The magnetic moment $\boldsymbol{\mu}$ has an interaction energy with a magnetic field $\boldsymbol{\mathcal{B}}$

$$E_m = -\boldsymbol{\mu} \cdot \boldsymbol{\mathcal{B}}. \tag{1}$$

The corresponding Stern–Gerlach force $\mathcal{F}_{SG}$ has been written in two formats

$$\mathcal{F}_{SG} \equiv \begin{cases} \nabla(\boldsymbol{\mu} \cdot \boldsymbol{\mathcal{B}}), & \text{Amperian Model,} \\ (\boldsymbol{\mu} \cdot \nabla)\,\boldsymbol{\mathcal{B}}, & \text{Gilbertian Model.} \end{cases} \tag{2}$$

(Special) Relativity evolves

Book 2017 link.springer.com/book/10.1007%2F978-3-319-51231-0

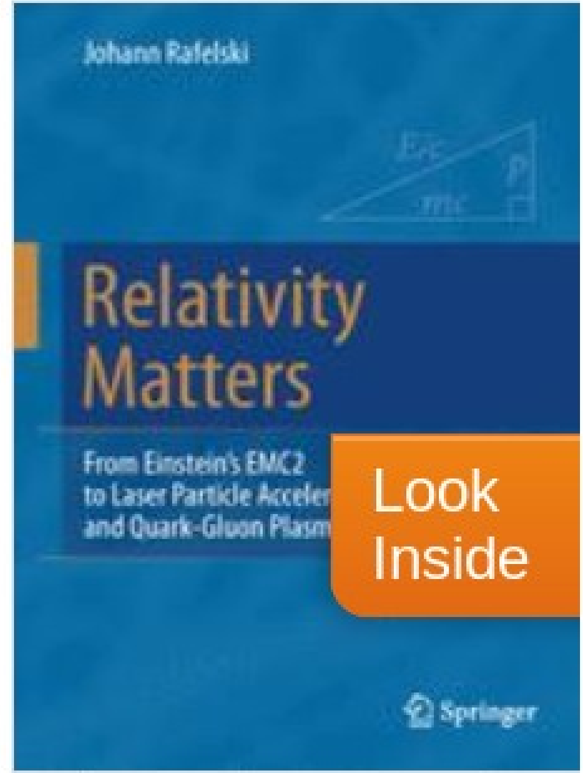
Relativity Matters

From Einstein's EMC2 to Laser Particle Acceleration and Quark-Gluon Plasma

Authors: Johann Rafelski

ISBN: 978-3-319-51230-3 (Print) 978-3-319-51231-0 (Online)

Springer Link



Text pdf available for free if your library subscribes to Springer Physics

Issues in Learning Special Relativity

Professors: If and when we need to say “paradox”, “not real”, it means we are not sure what we are teaching

Students: choose SR sources carefully, lots of bad stuff around (many false prophets)

Remember: “S” R bigger unfinished theory compared to GR and yet GR in minds of many superseeds SR

Message: insist SR “incomplete” as it is unfinished (acceleration)

Incomplete explanations: body contraction, time dilation, Doppler effect

Non-static context: evolving SR concepts cannot be presented 1905 way

Teaching SR I ask students about body contraction: I offer a choice -

What is "Lorentz contraction": $\gamma = 1/(1 - v^2/c^2)^{1/2}$

Some say space is contracted. Can this be true?

Other say this is distance contraction. What is this?

A few claim this is "apparent" body contraction. Apparent?

Einstein wrote a "response" in 1911 explaining that his and Lorentz views in this matter agree: body contraction is real (just like kinetic energy and momentum of a car is real even if it is zero for the driver, jr). In 1911 nobody would confound material body and space-time. That was before G_{ravity}R. GR short circuits SR thinking.

Relativity

Einstein 1905: Inertial Motion

Problems with understanding of body contraction, time dilation, Doppler effect: Frequent confounding of body behavior with coordinate transformation of space and time

Lorentz-Bell simpler: transfer bodies from on to another frame of reference using sub-nano-forces

1911 Einstein:

"...it (Lorentz-FitzGerald body contraction) is real and in principle observable by physical means by any non-comoving observer."

We understand the energy, momentum in this way

Spatial distance vs body length: Bell rockets

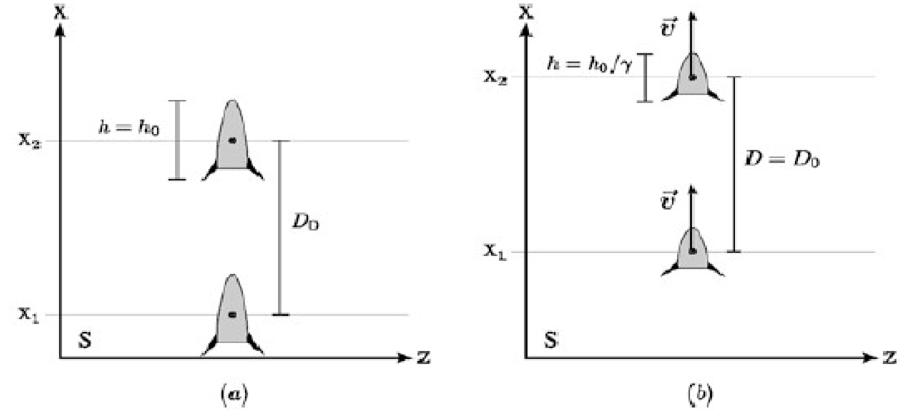


Fig. 10.2 Two rockets of length h separated by distance $D = x_2 - x_1 = D_0$. (a) at rest, and in case (b) moving at velocity $\vec{v}$ acquired at a later time

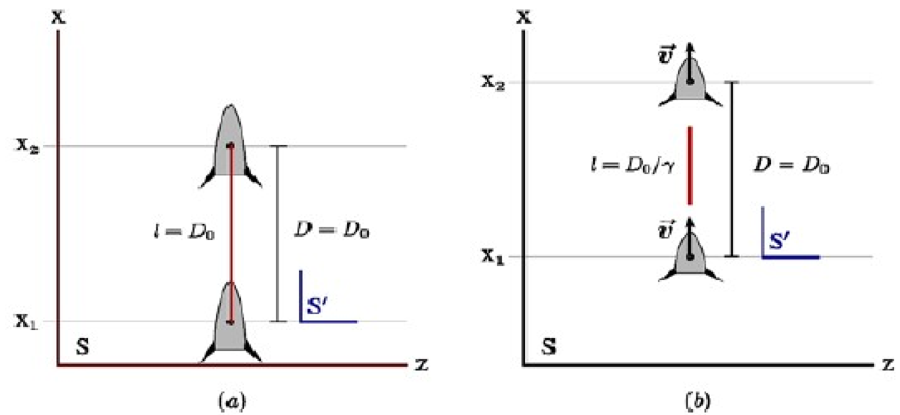


Fig. 10.3 Two rockets separated by distance $D = x_2 - x_1 = D_0$ and connected by a thin thread of (a) at rest, and in case (b) moving at velocity $\vec{v}$ acquired at a later time

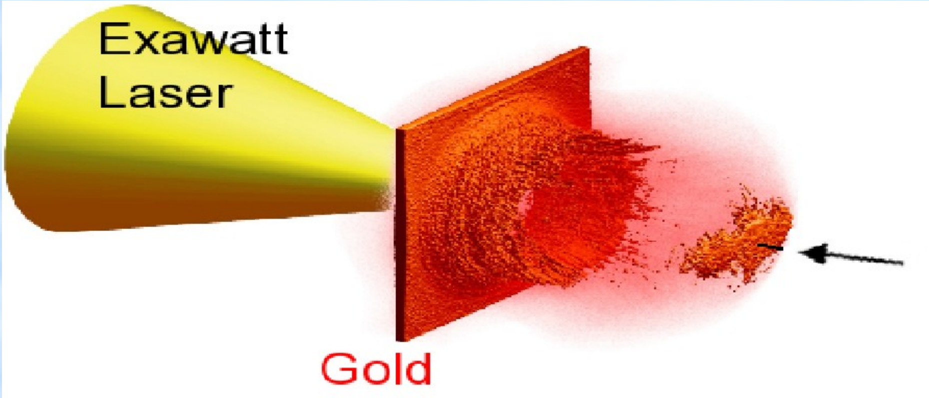
Is a passenger on a relativistic rocket aware she is “contracted”?

A. Einststein 1911: No - there is no absolute reference frame in the Universe, she cannot know against what she contracts.

J. S. Bell 1976 of “inequality fame”: advocates “physical reality” (Lorentz) view of relativity (idea not new): use accelerated motion to move from one inertial frame to another. The history of the shift between frames of reference allows to construct a “clock” for Lorentz contraction.

All of SR tested but body contraction

Idea: use reflection from relativistic electron mirror



Eur. Phys. J. A (2018) **54**: 29
DOI 10.1140/epja/i2018-12370-4

The moving electron cloud mirror is body compressed.

Johann Rafelski: Measurement of the Lorentz-FitzGerald Body Contraction

Body contraction experiment. — To accomplish our goal to build a laboratory-sized experiment we consider an ultra-intense ultra-short laser pulse shot at a thin (micron) foil. Such a pulse in its focal point can act as a micron-sized hammer pushing out of the foil an electron cloud accelerated to ultrarelativistic motion with a high value of Lorentz-factor γ_e. The emerging electron cloud compared to the original foil thickness will be Lorentz-FitzGerald compressed by γ_e.

A moving electron cloud acts as a relativistic mirror for a low intensity laser light bounce. The capability of the ultrarelativistic mirror to function depends on the electron cloud density; laser light can scatter coherently from a sufficiently high density cloud – what is low and high density is determined by comparing mean electron separation to the light wavelength.

two Lorentz transforms, first into the rest-frame of the mirror and upon reversal of the propagation direction of the light motion, transform back to the laboratory frame.

arXiv.org > physics > arXiv:1708.05670 EPJA, dedicated to memory of Walter Greiner

Relativity

A Einstein 1905: considers inertial Motion

a consistent framework, HOWEVER:

1905 “Special”
Relativity works since:
lab acceleration negligible:
“nano-forces” Theory
Incomplete: <u>missing</u> EM forces

Fundamental Problems with

Forces: **F**=e(**E**+**v** x **B**) “Lorentz”

multiple extenstions needed

1916 Einstein
Included Force of
Gravity by allowing
curved space-time
NOT a topic of today

- GR : consistent

“General” Relativity=
Gravity Relativity

Acceleration=0 approximation
how big is “a” in laboratory?

Ultra-relativistic electron in a magnet of 4.41Tesla

$a_{MAX}=(e/M_e)$ **vxB**

$=1.6\ 10^{-19}\ 3\ 10^{8}\ 4.41\ /(9.11\ 10^{-31})=$**$2.33\text{x}10^{20}$m/s^2=nano a_{cr}**

Compare: Natural “unit-1” acceleration

a_{cr} $=M_ec^2\,c/(h/2\pi)=9.11\ 10^{-31}\ 27\ 10^{24}/1.05\ 10^{-34}=$**$2.33\ 10^{29}$m/s^2**

This is also the acceleration generated by “critical” or Schwinger EM fields”: $E_{cr}=(M_ec^2)^2/(ehc/2\pi)=1.323\ 10^{18}$ V/m

$B_{cr}=(M_ec^2)^2/(ehc^2/2\pi)=4.414\ 10^{9}$ T

But: Does Acceleration Exist?

- Acceleration not inherent to quantum mechanics: all quantum operators made of x,p

Gravity as deformation of space time geometry: motion on geodetics (generalized straight lines)

However: A classical “charged” accelerated particle radiates demonstrating it “knows” when in state of accelerated motion. How is “know” possible?

Mach’s Principle: Acceleration REQUIRES as reference a (set of equivalent) inertial frame(s) so we know a body is accelerated. This path leads back to the aether.

Acceleration & Mach's Principle

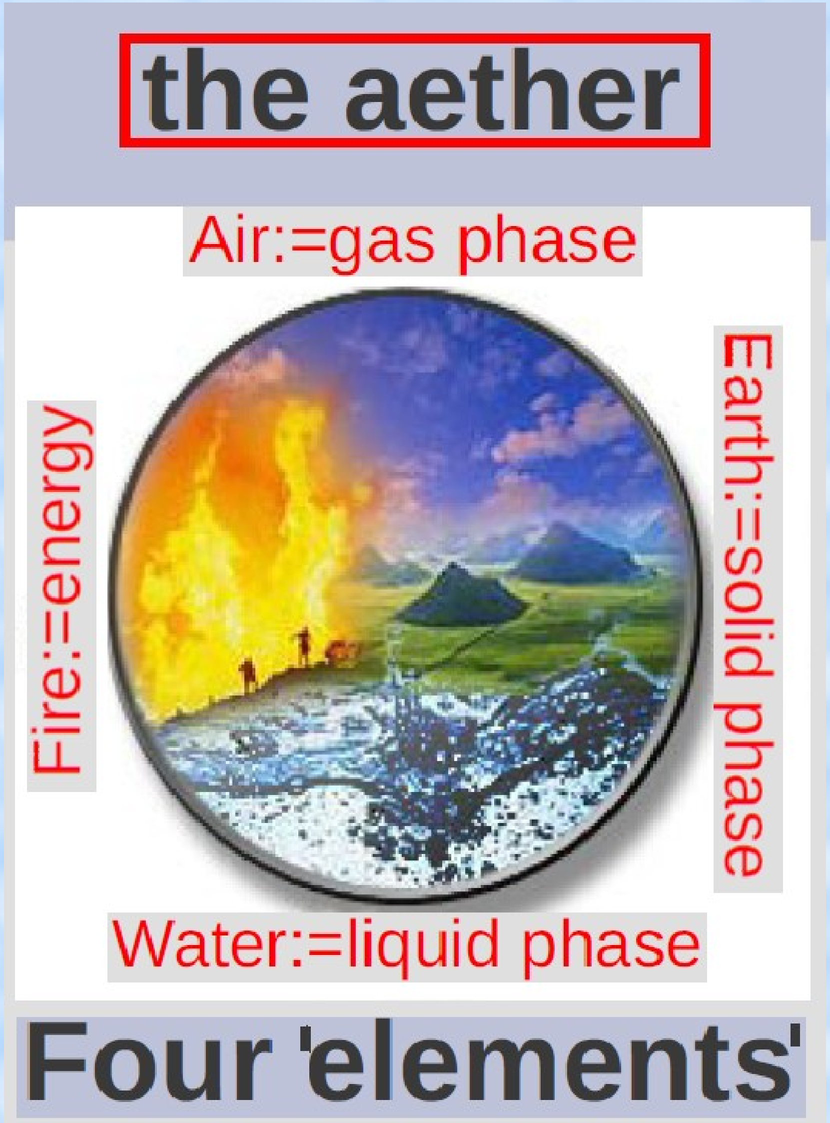
Ernst Mach
1838-1916
In Prague
1868-1895

Recognition/measurement of accleration requires a Reference frame: what was once the set of fixed stars in the sky can be today CMB photon freeze-out reference frame. However, we define acceleration locally -- the quantum vacuum allows us to know about acceleration. To be consistent with special relativity: all inertial observers form an equivalence class. Quantum vacuum transparent to inertial motion, resists acceleration: radiation friction.

In Einstein's gravity relativity the reference frame was provided by the metric. However, there is no "acceleration", a dust of gravitating particles is in free fall.

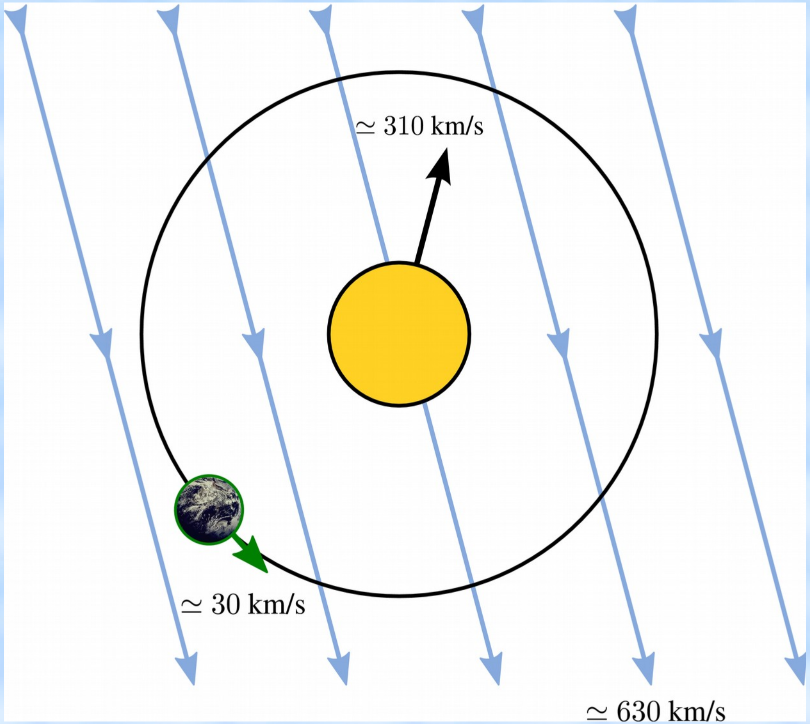
***... with the new theory of electrodynamics* (QED, JR) *we are rather forced to have an aether.* – P.A.M. Dirac, 'Is There an Aether?,' *Nature*, v.168, 1951, p.906.**

Exchange of letters with a few, including L. Infeld shows that nobody understood Dirac and Dirac did not quite understand that he was right about QED=aether.



The word aether in Homeric Greek means “pure, fresh air” or “clear sky”, pure essence where the gods lived and which they breathed. The aether was believed in ancient and medieval science to be the substance that filled the region of the universe above the terrestrial sphere. Aristotle imposed aether as a fifth element filling all space. Aether was later called **quintessence** (from quinta essentia, "fifth element"). The **"luminiferous aether"** (light carrying aether) is the “substance” believed by Maxwell, Larmor, Lorentz to permeate all the Universe. Einstein flips on the topic, introduces **relativistic aether 1920.**

Michelson-Morley: No aether wind, no drag



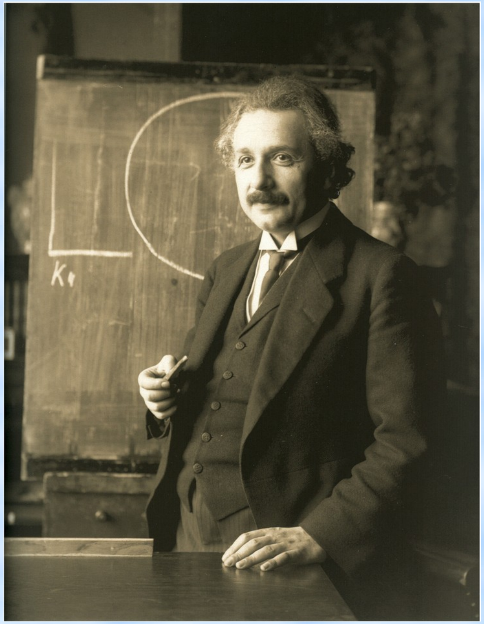
- The Earth moves in space (today we know the speed with reference to the big-bang frame of reference). Michelson-Morley experiment: no aether dragged along, birth of Lorentz-Fitzgerald contraction and relativity.
- **Einstein 1905:** who needs aether? All inertial observers are equivalent (principle of relativity).

Einstein's view about aether changes drastically by 1920

How can the laws of physics be known in all Universe?

"Recapitulating, we may say that according to the general theory of relativity space is endowed with physical qualities; in this sense, therefore, there exists an aether. But this aether may not be thought of as endowed with the quality characteristic of **ponderable media**, as consisting of parts which may be tracked through time. The idea of motion may not be applied to it.

"According to the general theory of relativity space without aether is unthinkable; for in such space there not only would be no propagation of light, but also no possibility of existence for standards of space and time (measuring-rods and clocks), nor therefore any space-time intervals in the physical sense."



Albert Einstein, *Ather und die Relativitaetstheorie* (Berlin, 1920):

TODAY: The laws of physics are **encoded** in quantum vacuum structure

We need to extend (S)R to account for missing forces:

1: EM Radiation reaction force

High Energy Physics - Phenomenology arXiv.org > hep-ph > arXiv:1005.3980

Phys.Rev.D82:096012,2010
10.1103/PhysRevD.82.096012

Effects of Radiation-Reaction in Relativistic Laser Acceleration

Y. Hadad, L. Labun, J. Rafelski, N. Elkina, C. Klier, H. Ruhl

(Submitted on 21 May 2010 (v1), last revised 16 Nov 2010 (this version, v3))

The goal of this paper is twofold: to explore the response of classical charges to electromagnetic force at the level of unity in natural units and to establish a criterion that determines physical parameters for which the related radiation-reaction effects are detectable. In pursuit of this goal, the Landau-Lifshitz equation is solved analytically for an arbitrary (transverse) electromagnetic pulse. A comparative study of the radiation emission of an electron in a linearly polarized pulse for the Landau-Lifshitz equation and for the Lorentz force equation reveals the radiation-reaction dominated regime, in which radiation-reaction effects overcome the influence of the external fields. The case of a relativistic electron that is slowed down by a counter propagating electromagnetic pulse is studied in detail. We further show that when the electron experiences acceleration of order unity, the dynamics of the Lorentz force equation, the Landau-Lifshitz equation and the Lorentz-Abraham-Dirac equation all result in different radiation emission that could be distinguished in experiment. Finally, our analytic and numerical results are compared with those appearing in the literature.

Radiation-Acceleration Trouble

Conventional SR+Electromagnetic theory is incomplete: radiation emitted needs to be incorporated as a back-reaction "patch":

1) **Inertial Force = Lorentz-force**-->get world line of particles=source of fields
2) **Source of Fields = Maxwell fields** --> get fields, and omit radiated fields
3) **Fields fix Lorentz force --> go to 1**.

So long as radiated fields are small, we can modify the Lorentz Force to account for radiated field back reaction approximately

458 29 Afterword: Acceleration

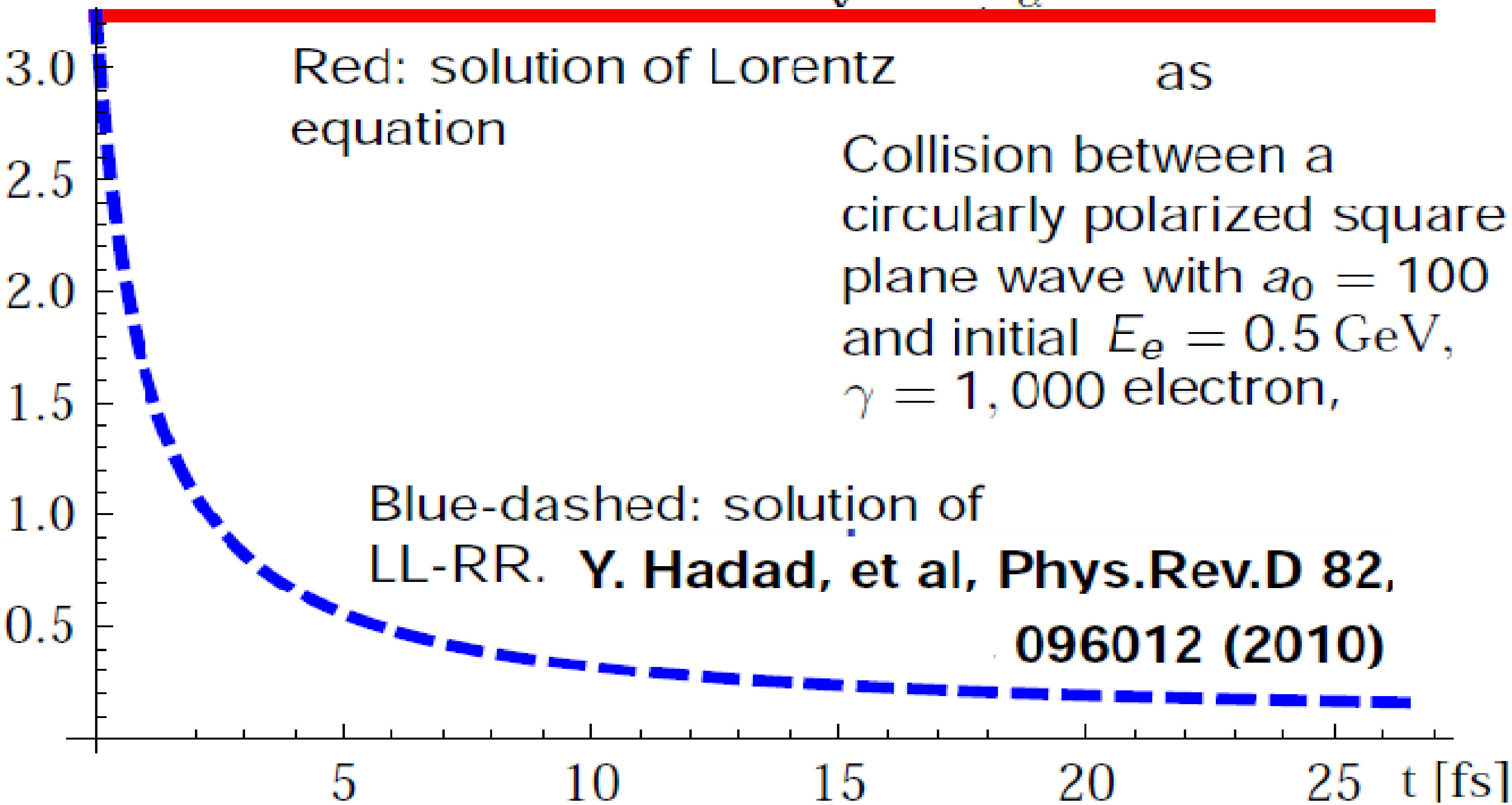
Table 29.1 Models of radiation reaction extensions of the Lorentz force

Maxwell-Lorentz	$\mathbf{m}\dot{\mathbf{u}}^{\mu} = \mathbf{e}\mathbf{F}^{\mu\nu}\mathbf{u}_{\nu}$
LAD[33]	$\mathbf{m}\dot{\mathbf{u}}^{\mu} = \mathbf{e}\mathbf{F}^{\mu\nu}\mathbf{u}_{\nu} + m\tau_0\left[g^{\mu\nu} - \frac{u^{\mu}u^{\nu}}{c^2}\right]\ddot{u}_{\nu}\,,\ \tau_0 = \frac{2}{3}\frac{e^2}{4\pi\epsilon_0 mc^3}$
Landau-Lifshitz[35]	$\mathbf{m}\dot{\mathbf{u}}^{\mu} = \mathbf{e}\mathbf{F}^{\mu\nu}\mathbf{u}_{\nu} + e\tau_0\left\{u^{\gamma}\partial_{\gamma}F^{\mu\delta}u_{\delta} + \frac{e}{m}\left(g^{\mu\gamma} - \frac{u^{\mu}u^{\gamma}}{c^2}\right)F_{\gamma\beta}F^{\beta}_{\delta}u^{\delta}\right\}$
Caldirola[36]	$0 = \mathbf{e}\mathbf{F}^{\mu\nu}(\tau)\mathbf{u}_{\nu}(\tau) - m\left[g^{\mu\nu} - \frac{u^{\mu}(\tau)u^{\nu}(\tau)}{c^2}\right]\frac{u_{\nu}(\tau) - u_{\nu}(\tau - 2\tau_0)}{2\tau_0}$

Solving LL Equation for a-crit

Example: Electron de-acceleration by a pulse

Lorentz invariant acceleration $\sqrt{-\dot{u}^\alpha \dot{u}_\alpha}$ $[m_e]$ function of time



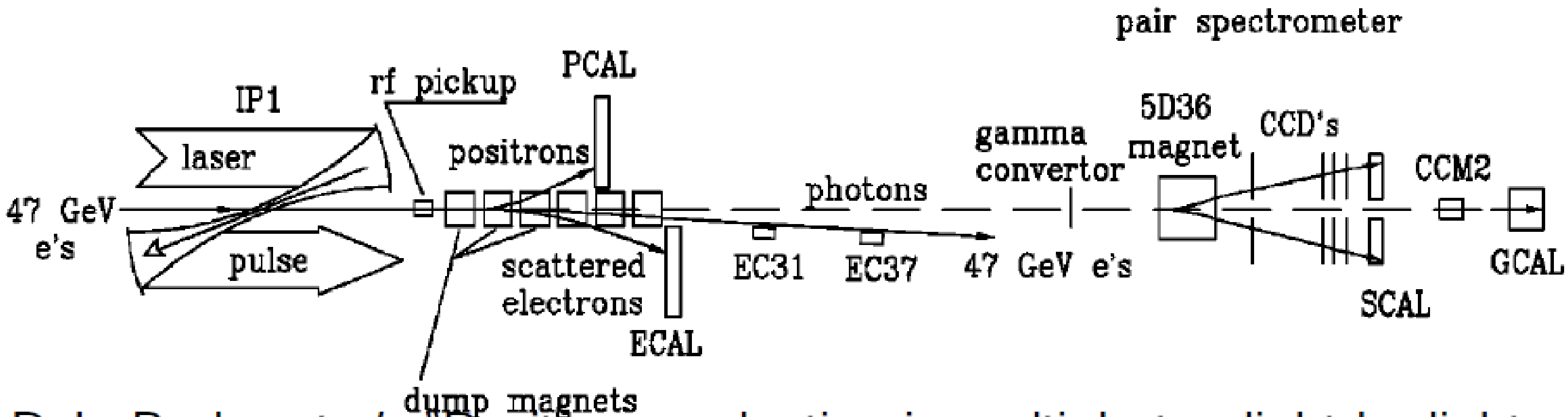
Probing EM-unit acceleration possible today

SLAC'95 experiment — *Proof of Principle*

$$p_e^0 = 46.6 \text{ GeV; in 1996/7 } a_0 = 0.4, \quad \left|\frac{du^\alpha}{d\tau}\right| = .073[m_e] \text{ (Peak)}$$

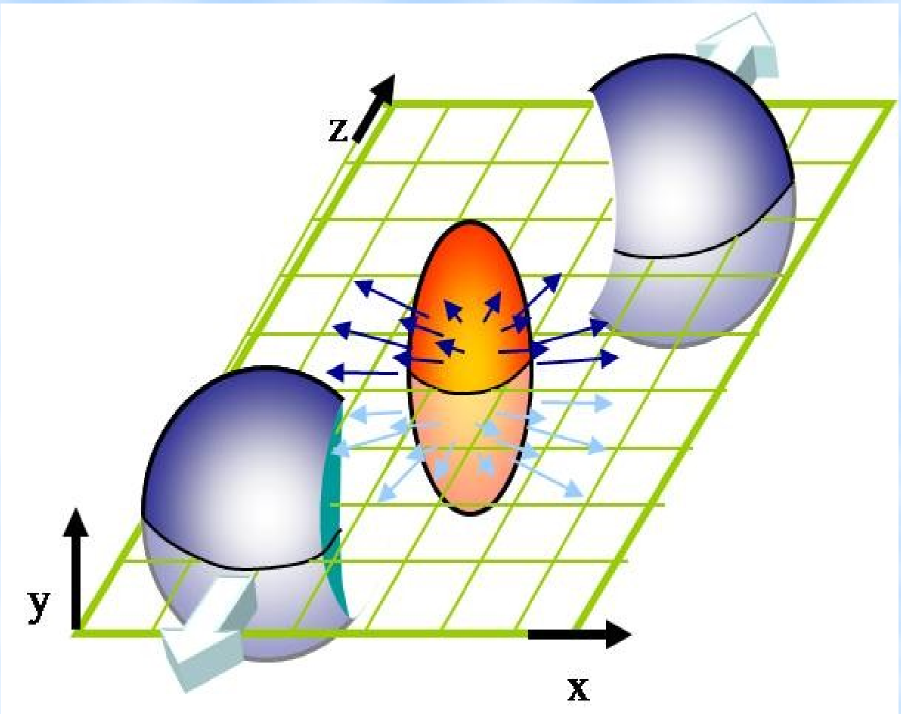
Multi-photon processes observed:

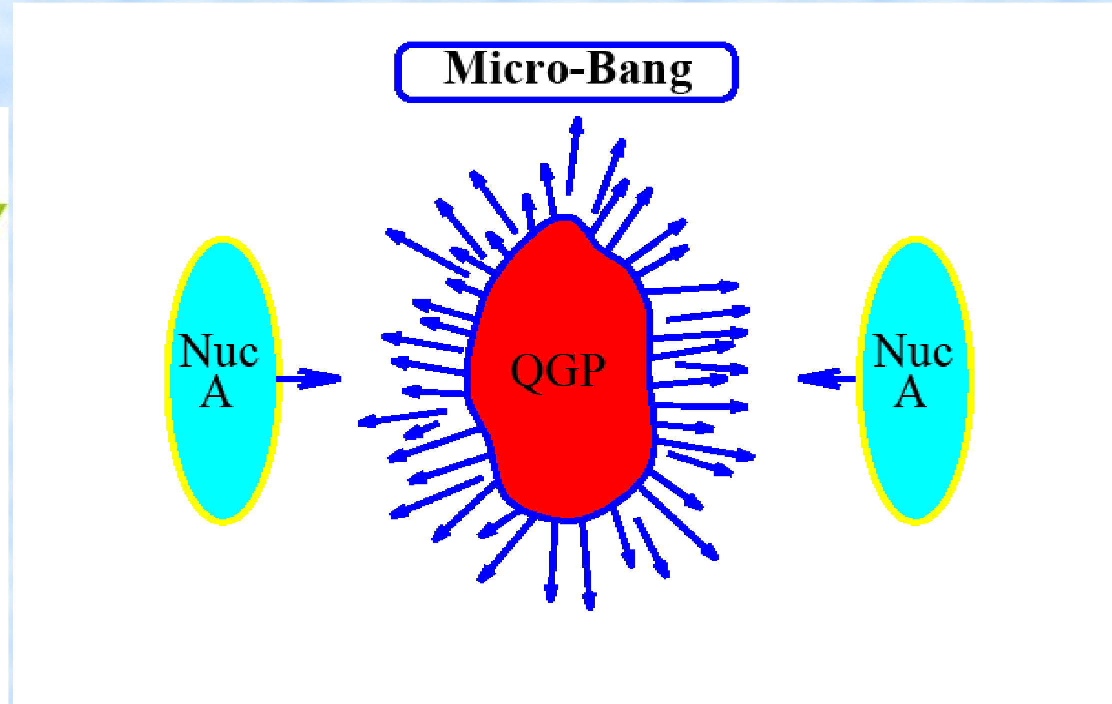
- Nonlinear Compton scattering
- Breit-Wheeler electron-positron pairs



- D. L. Burke *et al.*, "Positron production in multiphoton light-by-light scattering," Phys. Rev. Lett. **79**, 1626 (1997)
- C. Bamber *et al.*, "Studies of nonlinear QED in collisions of 46.6 GeV electrons with intense laser pulses" Phys. Rev. D **60**, 092004 (1999).

Another context for critical acceleration experiments: Relativistic Nuclear Collisions

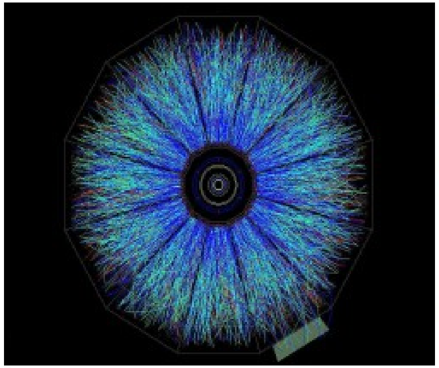




Big-Bang	Micro-Bang
$\tau \simeq 10\mu s$	$\tau \simeq 4\ 10^{-23}s$
$N_b\ /\ N \simeq 10^{-10}$	$N_b\ /\ N \simeq 0.1$

Nuclear Collisions at energy E>>Mc^2

Unit Acceleration in Strong Interactions



Two nuclei smashed into each other at highest achievable energy: components can be stopped in CM frame within $\Delta\tau \simeq 1$ fm/c. Tracks show multitude of particles produced, as seen at RHIC (BNL) and at CERN.

- The acceleration a required to stop some/any of the components of the colliding nuclei in CM: $a \simeq \frac{\Delta y}{M_i \Delta\tau}$. Full stopping: $\Delta y_{\mathrm{SPS}} = 2.9$, $\Delta y_{\mathrm{RHIC}} = 5.4$, larger at CERN. Considering constituent quark masses $M_i \simeq M_N/3 \simeq 310$ MeV we need $\Delta\tau_{\mathrm{SPS}} < 1.8$ fm/c and longer times at colliders to exceed critical a.

- The soft electromagnetic radiation in hadron reactions (*A. Belognni et al. [WA91 Collaboration], "Confirmation of a soft photon signal in excess of QED expectations in π–p interactions at 280-GeV/c," Phys. Lett. B* **408**, *487 (1997) [arXiv:hep-ex/9710006].*) and heavy ion reactions exceeds the perturbative QED predictions significantly

Missing EM “Stern-Gerlach” force

Eur. Phys. J. C (2018) 78:6
https://doi.org/10.1140/epjc/s10052-017-5493-2
THE EUROPEAN PHYSICAL JOURNAL C
Regular Article - Theoretical Physics

Physics > Classical Physics
arXiv.org > physics > arXiv:1712.01825

Relativistic Dynamics of Point Magnetic Moment

Johann Rafelski, Martin Formanek, Andrew Steinmetz

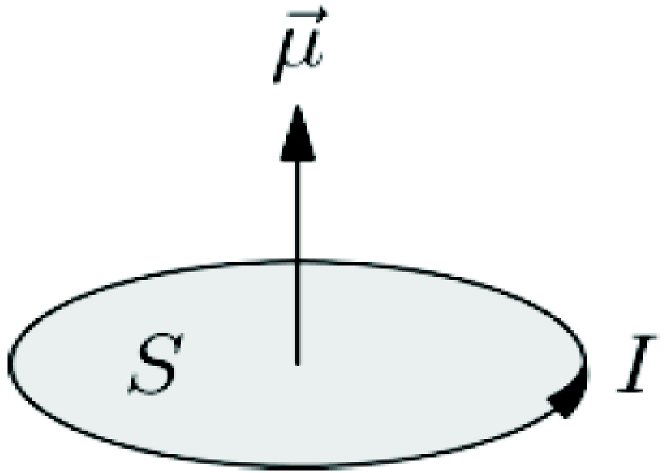
(Submitted on 1 Dec 2017)

The covariant motion of a classical point particle with magnetic moment in the presence of (external) electromagnetic fields is revisited. We are interested in understanding Lorentz force extension involving point particle magnetic moment (Stern-Gerlach force) and how the spin precession dynamics is modified for consistency. We introduce spin as a classical particle property inherent to Poincare\'e symmetry of space-time. We propose a covariant formulation of the magnetic force based on a \lq magnetic\rq\ 4-potential and show how the point particle magnetic moment relates to the Amperian (current loop) and Gilbertian (magnetic monopole) description. We show that covariant spin precession lacks a unique form and discuss connection to $g-2$ anomaly. We consider variational action principle and find that a consistent extension of Lorentz force to include magnetic spin force is not straightforward. We look at non-covariant particle dynamics, and present a short introduction to dynamics of (neutral) particles hit by a laser pulse of arbitrary shape.

Two models for magnetic dipole Stern-Gerlach force

All agree: magnetic potential $\boldsymbol{U} = -\boldsymbol{\mu} \cdot \boldsymbol{\mathcal{B}}$

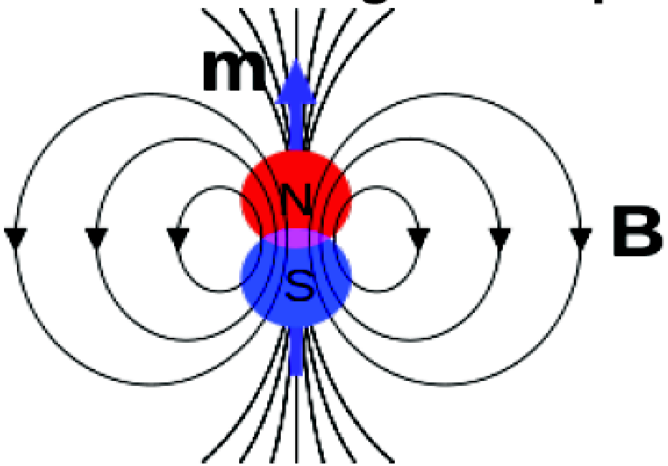
Amperian - current loop



en.wikipedia.org/wiki/Magnetic_moment

$$\mathcal{F}_{\text{ASG}} = \boldsymbol{\nabla}(\boldsymbol{\mu} \cdot \boldsymbol{\mathcal{B}})$$

Gilbertian - magnetic dipole



en.wikipedia.org/wiki/Magnetic_dipole

$$\mathcal{F}_{\text{GSG}} = (\boldsymbol{\mu} \cdot \boldsymbol{\nabla})\boldsymbol{\mathcal{B}}$$

Named after William Gilbert 1544-1603

There are no magnetic monopoles. Point particles have no current loops. We need a third model

The new model should

- Apply to magnetic moment of point-spinning (classical) particle;
- Lead to one force-type only unifying Amperian and Gilbertian forms as equivalent;
- Be consistent in form with torque and spin dynamics:

Definition of torque:

$\boldsymbol{\tau} = \boldsymbol{\mu}_{\mathrm{T}} \times \boldsymbol{\mathcal{B}}$ Is $\boldsymbol{\mu}_{\mathrm{T}}$ same as $\boldsymbol{\mu}$?

- We want forces, torque to be in covariant Relativity format, that is we seek an extension of the 'Lorentz-Force'

Lorentz Force: EM-Fields $F^{\mu\nu}$, 4-velocity u_ν

$$\frac{du^\mu}{d\tau} = \frac{e}{m} F^{\mu\nu} u_\nu ,$$

s^ν: classical Spin of point particle

Non-rotating 'spin' natural in quantum Dirac equation; this that doesn't mean that spin is a quantum property! Spin arises in the context of Minkowski space-time symmetry transformations: **Poincaré group**. There are two **Casimir operators** commuting with all 10 generators

$$\bar{C}_1 \equiv \bar{p}_\mu \bar{p}^\mu; \quad C_1 = m^2 c^2 \qquad \bar{C}_2 \equiv \bar{w}_\mu \bar{w}^\mu; \quad \bar{s}^\mu \equiv \frac{\bar{w}^\mu}{\sqrt{C_1}}$$

$\bar{w}^\mu$ is axial Pauli-Lubanski 4-vector made out of generators of rotations $\overline{\boldsymbol{J}}$ and boosts $\overline{\boldsymbol{K}}$

$$\bar{w}_\mu = \overline{M}^*_{\mu\nu} \bar{p}^\nu, \quad \overline{M}^*_{\mu\nu} = \begin{pmatrix} 0 & -\overline{J}_1 & -\overline{J}_2 & -\overline{J}_3 \\ \overline{J}_1 & 0 & -\overline{K}_3 & \overline{K}_2 \\ \overline{J}_2 & \overline{K}_3 & 0 & -\overline{K}_1 \\ \overline{J}_3 & -\overline{K}_2 & \overline{K}_1 & 0 \end{pmatrix} \Rightarrow \boxed{\bar{u}_\mu \bar{s}^\mu = \frac{\bar{p}_\mu}{m} \bar{s}^\mu = 0}$$

Any and each point particle belongs to an irreducible representation of the Poincare group described by the eigenvalues C_1 and C_2 of the Casimir operators. $\sqrt{C_1}$ relates to mass and $\sqrt{C_2/C_1} \equiv |s^\mu|$ to spin.

Relativistic ‘magnetic potential’

Since $E_{\rm mag} = -\boldsymbol{\mu}\cdot\boldsymbol{\mathcal{B}} \equiv U^0_{\rm mag}$

We look at a magnetic 4-potential B^μ akin to e-4-potential A^μ

$$B_\mu \equiv F^*_{\mu\nu}s^\nu, \quad F^*_{\mu\nu} \equiv \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}F^{\alpha\beta}, \quad F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu$$

since s_μ is axial, B^μ is a polar 4-vector. In the rest frame of the particle

Need magnetic ‘charge’ d

$$U^0_{\rm mag} = B^0 cd = -\boldsymbol{\mu}\cdot\boldsymbol{\mathcal{B}}, \quad \boldsymbol{s}\, dc = \boldsymbol{\mu}$$

B^μ generates additional magnetic force

$$m\frac{du^\mu}{d\tau} \equiv F^\mu_{\rm ASG} = (eF^{\mu\nu} + G^{\mu\nu}d)u_\nu, \quad G^{\mu\nu} \equiv \partial^\mu B^\nu - \partial^\nu B^\mu .$$

Covariant Amperian and Gilbertian Stern-Gerlach force

The magnetic force will be now identified to be the Amperian form:

ASG force and the rest frame of a particle

$$F^\mu_{\rm ASG} = eF^{\mu\nu}u_\nu - u\cdot\partial F^{\star\,\mu\nu}s_\nu\, d + \partial^\mu(u\cdot F^\star\cdot s\, d)$$

$$F^\mu_{\rm ASG}|_{\rm RF} = \left\{0,\ e\boldsymbol{\mathcal{E}} + \nabla(\boldsymbol{\mu}\cdot\boldsymbol{\mathcal{B}}) - \frac{1}{c^2}\,\boldsymbol{\mu}\times\frac{\partial\boldsymbol{\mathcal{E}}}{\partial t}\right\}$$

Another approach that allows us to find the Gilbertian force:

We try to modify the fields

$$eF^{\mu\nu} \rightarrow \boxed{\widetilde{F}^{\mu\nu} = eF^{\mu\nu} - s\cdot\partial F^{\star\,\mu\nu}\, d}\ ,$$

ASG=GSG force and the rest frame of a particle

$$F^\mu_{\rm ASG} = F^\mu_{\rm GSG} = (eF^{\mu\nu} - s\cdot\partial F^{\star\,\mu\nu}\, d)\, u_\nu - \mu_0 j^\gamma \epsilon_{\gamma\alpha\beta\nu}u^\alpha s^\beta g^{\nu\mu}\, d$$

$$F^\mu_{\rm GSG}|_{\rm RF} = \{0,\ e\boldsymbol{\mathcal{E}} + (\boldsymbol{\mu}\cdot\nabla)\boldsymbol{\mathcal{B}} + \mu_0\boldsymbol{\mu}\times\boldsymbol{j}\}$$

Equivalence of point particle magnetic moment forces

Based on this we can write two equivalent generalizations of the Lorentz force

ASG, GSG: two ways to write one and the same thing

$$F^{\mu} = F^{\mu}_{\mathrm{ASG}} = eF^{\mu\nu}u_{\nu} - u \cdot \partial\, F^{\star\,\mu\nu} s_{\nu}\, d + \partial^{\mu}(u \cdot F^{\star} \cdot s\, d)$$

$$F^{\mu} = F^{\mu}_{\mathrm{GSG}} = (eF^{\mu\nu} - s \cdot \partial\, F^{\star\,\mu\nu}\, d)\, u_{\nu} - \mu_0 j^{\gamma} \epsilon_{\gamma\alpha\beta\nu} u^{\alpha} s^{\beta} g^{\nu\mu}\, d$$

$\boldsymbol{\nabla}(\boldsymbol{\mu} \cdot \boldsymbol{\mathcal{B}}) - (\boldsymbol{\mu} \cdot \boldsymbol{\nabla})\boldsymbol{\mathcal{B}} = \boldsymbol{\mu} \times (\boldsymbol{\nabla} \times \boldsymbol{\mathcal{B}})$ with this we obtain

In rest frame

$$\begin{aligned} 0 &= [\boldsymbol{F}_{\mathrm{ASG}} - \boldsymbol{F}_{\mathrm{GSG}}]_{\mathrm{RF}} \\ &= \boldsymbol{\mu} \times \left(-\frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t} + \boldsymbol{\nabla} \times \boldsymbol{B} - \mu_0 \boldsymbol{j} \right) = 0\,. \end{aligned}$$

We recognize Maxwell equation in parenthesis

How the modified force generates new spin dynamics (torque)

J. S. Schwinger, *"Spin precession: A dynamical discussion"*, American Journal of Physics **42**, (1974) 510,

Schwinger shows how the TMBT spin dynamics relates to EM force: given $u \cdot s = 0$ he takes proper time τ derivative $\dot{u} \cdot s + u \cdot \dot{s} = 0$ and substituting force for $\dot{u}$ for the case of Lorentz dynamics he argues:

$$u_\mu \left(\frac{ds^\mu}{d\tau} - \frac{e}{m} F^{\mu\nu} s_\nu \right) = 0 \, .$$

The general solution satisfying this equation is

$$\frac{ds^\mu}{d\tau} = \frac{e}{m} F^{\mu\nu} s_\nu + \frac{\widetilde{a}e}{m} \left(F^{\mu\nu} s_\nu - \frac{u^\mu}{c^2} (u \cdot F \cdot s) \right)$$

We repeat the same for our generalized Lorentz force: each component $F^{\mu\nu}$ and $G^{\mu\nu}$ induces two independent integration constants ($\widetilde{a}$ and $\widetilde{b}$ below)

Covariant dynamical equations

From now on we use the Gilbertian form of the Lorentz force F^{μ}_{GSG} in vacuum $j^{\mu} = 0$.
The dynamical 'Schwinger' spin equation is obtained as described above

Coupled covariant motion of particle 4-velocity u^{μ} and spin s^{μ}

$$\frac{du^{\mu}}{d\tau} = \frac{1}{m}(eF^{\mu\nu} - s \cdot \partial F^{*\mu\nu} d)u_{\nu}$$

$$\frac{ds^{\mu}}{d\tau} = \frac{1+\widetilde{a}}{m}\left(eF^{\mu\nu} - \frac{1+\widetilde{b}}{1+\widetilde{a}} s \cdot \partial F^{*\mu\nu} d\right) s_{\nu} - \widetilde{a}\frac{u^{\mu}}{mc^2}\left(u \cdot \left(eF - \frac{\widetilde{b}}{\widetilde{a}} s \cdot \partial F^{*} d\right) \cdot s\right)$$

- $\widetilde{a}$ and $\widetilde{b}$ are arbitrary integration constants
- Reduces to TBMT equations for $d = 0$ with $\widetilde{a} \to a$
- Dynamics of a neutral particle depends only on $\widetilde{b}$

Particle in an external plane wave (laser) field

Or: is it possible using lasers to guide neutral particles?
Plane wave field with profile function f has the 4-potential

$$A^\mu(\xi) = a_0 \varepsilon^\mu f(\xi), \quad \xi = k \cdot x, \quad k \cdot \varepsilon = 0, \quad k^2 = 0$$

Squaring the generalized Lorentz force equation gives us a formula for invariant acceleration

$$\dot{u}^2 = -\left[\left(\frac{e\, a_0}{mc^2} f'(\xi)\right)^2 + \left(k \cdot s\, f''(\xi) \frac{a_0\, d}{mc^2}\right)^2\right] (ck \cdot u(0))^2$$

Prime denotes derivative with respect to the phase ξ.
$(k \cdot s(\tau))$ must be obtained integrating dynamical equations.
$k \cdot u(0) = c\omega\gamma_0(1 - \boldsymbol{\beta_0} \cdot \boldsymbol{n})$, a fancy way to write the Doppler factor.
Since $k^2 = 0$, s^2 fixed, $s \cdot u = 0$ we find $k \cdot s$ is bounded for any γ.
Particle motion depends on effective Doppler shifter force it sees.

Conclusions

After many years of neglect we find ourselves already immersed into **an encore of SR** with opportunties in probing acceleration frontier in high intensity **laser-particle interaction** and **RHI experiments** probing critical acceleration. Teaching relativity to future researchers in this field in a way that prepares for these new opportunity presents a challenge. Novel challenges arise when Lorentz force is replaced to account for acceleration friction and as described, magnetic moment force.