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2. Formation and Observation of the Quark-Gluon Plasma*

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2.1. Overview

What purpose could we follow when arguing for the study of high energy nuclear collisions [1]? It would appear that the complexity of such collisions, involving several hundreds of valence quarks, must cover up all the interesting feature of fundamental interactions. I would like to argue in this report that much in the nature and properties of strong interactions can be studied by creating in the laboratory a new state of matter – the quark-gluon plasma [2]. Unlike hadron–hadron collisions we anticipate that, in an important fraction of nucleus–nucleus collisions, each participating quark will scatter many times before joining into an asymptotic hadronic state. The associated simplification of the involved physics arises because we can use in such a case the well established methods of statistical physics in order to connect the microscopic world with effects and properties visible to experimentalists' eyes. Alone the presumption of an approximate thermochemical equilibrium frees us from the dependence on details of quark wavefunctions in a small hadronic bag consisting of only few quarks.

There are several stages in this new and exciting field of high energy physics. The first one concerns the willingness to accept the fact that available energy is equipartitioned among accessible degrees of freedom. This means that there exists a domain in space, in which, in a proper Lorentz form, the energy of the longitudinal motion has been largely transformed to transverse degrees of freedom. We call this region "fireball". The physical variables characterising a fireball are: energy density, baryon number density and volume. The basic question concerns the internal structure of the fireball – it can consist either of individual hadrons or, instead, of quarks and gluons in a new physical phase: they look deconfined as they move freely over the volume of the fireball. It appears that the phase transition from the hadronic gas phase to the quark-gluon plasma is mainly controlled by the energy density of the fireball. Several estimates [2], lead to 0.6–1 $\mathrm{GeV/fm^3}$ for the critical energy density, to be compared with a value of 0.16 $\mathrm{GeV/fm^3}$ inside individual hadrons. Many theoretical questions about strong interactions will be settled if the parameters and nature of the phase transition are determined. We turn to these problems further below.

The second stage of the developments in this field concerns the interaction of the experimentalists with the plasma. It is quite difficult to insert a thermometer and to measure baryon density at $T = 150$ MeV and threefold or even higher nuclear compressions. We must either use only electromagnetically interacting particles [3] (photons, lepton pairs) in order to get them out of the plasma or study the heavy flavour abundance generated in the collision [4]. To obtain a better impression of what is meant imagine that strange quarks are very abundant in the plasma (and indeed they are!). Then, since a (sss)-state is bound and stable in the hot perturbative QCD-vacuum, it would be the most abundant

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baryon to emerge from the plasma. I doubt that such an "Omegaisation" of nuclear matter could leave any doubts about the occurrence of a phase transition. Other exotic hadrons [5] such as e.g. csq, c$\bar{s}$ etc. would also support this conclusion. But even the enhancement of the more accessible abundances of $\bar{\Lambda}$ may already be sufficient for our purposes.

But there is more to meet the eyes. Restoration of the perturbative QCD vacuum may be followed at higher and higher energy densities by restoration of chiral symmetry, as shown qualitatively in fig. 2.1, then by SU(2) symmetry (and finally by SU(5) symmetry!). If the fact that we can trace back the evolution of the universe [6] in the laboratory does not excite one's fantasy, one may then remember that the plasma state is the only place known (after the universe was created) where one can "burn" baryon number, thus releasing the energy from the Big Bang stored in matter. Perhaps sufficiently extreme conditions that are here necessary are "created" inside quasars, thus leading to the enormous energies radiated by these stellar objects.

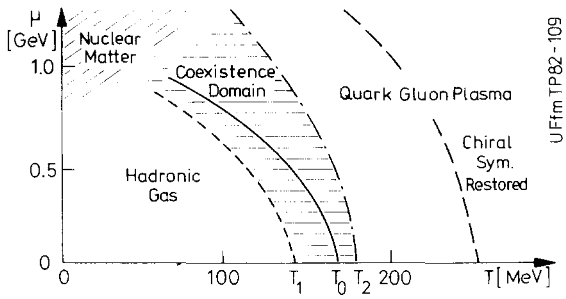

Fig. 2.1. Phase diagram of hadronic matter in the μ-T plane.

Coming back to earth we begin by recalling that in a statistical description of matter the unhandy microscopical variables: energy, baryon number etc. are replaced by thermodynamical quantities; the temperature T is a measure of energy per degree of freedom, the baryon chemical potential μ controls the mean baryon density: Statistical quantities such as entropy (measure of the number of accessible states), pressure, heat capacity etc. will be also functions of T and μ, to be determined. The theoretical techniques required for the description of both and quite different phases: the hadronic gas and the quark gluon plasma, must allow for the formation of numerous hadronic resonances on the one side [7], which then dissolve at sufficiently high spatial density in a state consisting of the fundamental constituents. At this point we must appreciate the importance and help provided by high temperature. To obtain high particle density we may, instead of compressing matter (which as it turns out is quite difficult), heat it up; many pions are easily generated, leading to the occurrence of a transition at moderate (even vanishing) baryon density [8].

2.2. Thermodynamics of interacting hadrons

The main hypothesis which allows one to simplify the situation is to postulate the resonance dominance of hadron–hadron interactions [7] – in this case the hadronic gas phase is practically a superposition of an infinity of different hadronic gases and all information about the interaction is hidden in the mass spectrum $\tau(m^2, b)$, which describes the number of hadrons of baryon number b in a mass interval dm^2 [9].

We survey in the following the developments discussed in refs. [8, 9]. We assume that the mass spectrum $\tau(m^2, b)$ is already known. The grand microcanonical level density is then given by an invariant phase space integral. The extreme richness of the spectrum $\tau(m^2, b) \sim \exp(m/T_0)$ enables us to neglect Fermi and Bose statistics above $T \approx 50$ MeV and to treat all particles as "Boltzmannions". We find:

$$\sigma(p, V_{\mathrm{ex}}, b) = \delta^4(p)\,\delta_{\mathrm{K}}(b) + \sum_{N=1}^{\infty} \frac{1}{N!}\,\delta^4\Big(p - \sum_{i=1}^{N} p_i\Big) \sum_{\{b_i\}} \delta_{\mathrm{K}}\Big(b - \sum_{i=1}^{N} b_i\Big) \prod_{i=1}^{N} \frac{2\Delta_\mu p_i^\mu}{(2\pi)^3}\,\tau(p_i^2, b_i)\,\mathrm{d}^4 p_i\,. \tag{1}$$

In this expression the first term corresponds to the vacuum state. The Nth term is the sum over all possible partitions of the total baryon number and of the total momentum p among N Boltzmannions, each having an internal number of quantum states given by $\tau(p_i^2, b_i)$. These Boltzmannions are hadronic resonances of baryon number b_1 $(-\infty < b_i < \infty)$. Every resonance can move freely in the remaining volume Δ left over from the external volume V_{ex}, after subtracting the proper volume V_{c} associated with all the hadrons:

$$\Delta^\mu := V_{\mathrm{ex}}^\mu - \sum_{i=1}^{N} V_{\mathrm{c},i}^\mu\,; \tag{2}$$

V^μ is a covariant generalisation of V_i. In the rest frame, we have $V_\mu = (V, 0)$.

In the generalisation (1) of the popular phase space formula, three essential features of hadronic interactions are now explicitly included:

(a) The dense set of hadronic resonances dominating particle scattering via $\tau(m_i^2, b_i)$.

(b) The proper natural volumes of hadronic resonances. This is done via Δ^μ.

(c) The conservation of baryon number and the clustering of hadrons into lumps of matter with $|b| > 1$.

The thermodynamic properties of the hot hadronic gas follow from the study of the grand partition function $Z(\beta, V\lambda)$, as obtained from the level density $\sigma(p, V, b)$, namely:

$$Z(\beta, V, \lambda) = \sum_{b=-\infty}^{\infty} \lambda^b \int \mathrm{e}^{-\beta\cdot p}\sigma(p, V, b)\,\mathrm{d}^4 p\,. \tag{3}$$

A covariant generalisation of thermodynamics, with an inverse temperature four vector β_μ has been used here. In the rest frame of the relativistic baryon chemical potential μ, we have:

$$\lambda = \exp(\mu/T)\,. \tag{4}$$

This is introduced in order to conserve baryon number in the statistical ensemble. All quantities of physical interest can then be derived as usual, differentiating $\ln Z$ with respect to its variables.

Eqs. (1–3) leave us with the task of finding the mass spectrum τ. Experimental knowledge of τ is limited to low excitations and/or to low baryon number. Following Hagedorn, we introduce here a theoretical model: "the statistical bootstrap", in order to obtain a mass spectrum consistent with direct (and indirect) experimental evidence. The qualitative arguments leading to an integral equation for $\tau(m^2, b)$ are the following: when V_{ex} in eq. (1) is just the proper volume V_{c} of a hadronic cluster, then σ in eq. (1), up to a normalization factor, is essentially the mass spectrum τ. Indeed, how could we distinguish between a composite system [as described by eq. (1)] compressed to the natural volume of a hadronic cluster and an "elementary" cluster having the same quantum numbers? We thus demand

$$\sigma(p, V, b)|_{V=V_c} \simeq H\tau(p^2, b) \tag{5}$$

where the "bootstrap constant" H is to be determined below. It is not simply sufficient to insert eq. (5) into eq. (1) to obtain the bootstrap equation for τ. More involved arguments are indeed necessary [8, 9] in order to obtain a "bootstrap equation" for the mass spectrum such as:

$$H\tau(p^2, b) = Hz_b\,\delta_0(p^2 - M_b^2) + \sum_{N=2}^{\infty} \frac{1}{N!} \int \delta^4\Big(p - \sum_{i=1}^{N} p_i\Big) \sum_{\{b_i\}} \delta_K\Big(b - \sum_{i=1}^{N} b_i\Big) \prod_{i=1}^{N} H\tau(p_i^2, b_i)\,\mathrm{d}^4p_i\,. \tag{6}$$

The first term is the lowest one-particle contribution to the mass spectrum, z_b is its statistical weight $(2I+1)(2J+1)$. The index "0" restricts the δ function to the positive root only. Only terms with $b = 0$, ± 1, corresponding to the lowest energy $q\bar{q}$ (pions) and qqq (nucleons) states contribute in the first term of eq. (6). All excitations are contained in the second term since an arbitrary quark constant can be achieved by combining $[(q\bar{q})^n(qqq)^m]$. Heavy flavours are ignored at this point but can easily be introduced. However they do not essentially influence the behaviour of τ. In the course of deriving the bootstrap equation (6) it turns out that the cluster volume V_c grows proportional to the invariant cluster mass [9]

$$V_c(p^2) = \sqrt{p^2}/(4B)\,. \tag{7}$$

The proportionality constant has been called $4B$ in order to establish a close relationship with the quark bag model [10]. The value of B can be derived from different considerations involving the true and perturbative QCD states. While the original MIT-bag fit gives $V^{1/4} = 145\,\mathrm{MeV}$, the most generally accepted value today is perhaps

$$B^{1/4} = 190\,\mathrm{MeV} \qquad \text{or} \qquad B = 170\,\mathrm{MeV/fm^3}\,. \tag{8}$$

The bootstrap constant H and the bag constant B are the only seemingly free parameters in this approach. As just pointed out, B is determined from other considerations, while H turns out to be inversely proportional to B. Hence, if one wishes to believe the last detail of the statistical bootstrap approach, there remains no free parameter in this approach. What this means for the transition from gas to plasma will be now shown.

Instead of solving eq. (6), which will lead us to the exponential mass spectrum [7],

$$\tau(m^2, b) \sim \exp(m/T_0) \tag{9}$$

we wish to concentrate here on the double integral (Laplace) transform of eq. (6) which will be all we need to establish the physical properties of the hadronic gas phase. Introducing the transforms of the one particle term, eq. (6)

$$\varphi(\beta, \lambda) := \sum_{b=-\infty}^{\infty} \lambda^b Hz_b\,\delta_0(p^2 - M_b^2)\,\mathrm{e}^{-\beta\cdot p}\,\mathrm{d}^4p \tag{10}$$

with pions and nucleons only

$$\varphi(\beta, \lambda) = 2\pi HT\Big[3mK_1\Big(\frac{m_\pi}{T}\Big) + 4\Big(\lambda + \frac{1}{\lambda}\Big)m_N K_1\Big(\frac{m_N}{T}\Big)\Big], \tag{11}$$

and of the mass spectrum:

$$\phi(\beta, \lambda) := \sum_{b=-\infty}^{\infty} \lambda^b \int H\tau(p^2, b)\, e^{-\beta\cdot p}\, d^4p. \tag{12}$$

We find for the entire eq. (6) the simple relation

$$\phi(\beta, \lambda) = \varphi(\beta, \lambda) + \exp[\phi(\beta, \lambda)] - \phi(\beta, \lambda) - 1. \tag{13}$$

To study the behaviour of $\phi(\beta, \lambda)$ we make use of the apparent implicit dependence:

$$\phi(\beta, \lambda) = G(\varphi(\beta, \lambda)) \tag{14a}$$

with function G being defined by eq. (13)

$$\varphi = 2G + 1 - \exp[G]. \tag{14b}$$

This function $G(\varphi)$ is shown in fig. 2.2. As is apparent there there is a maximal value φ_0

$$\varphi_0 = \ln(4/e) = 0.3863\ldots, \tag{14c}$$

beyond which the function G has no real solutions. Recalling the physical meaning of G, eqs. (14a, 12), we conclude that eq. (14c) establishes a boundary in the λ (i.e. μ), T plane beyond which the hadronic gas phase cannot exist. This boundary is implicitly given by the relation (11):

$$\ln(4/e) = 2\pi HT_{cr}[3m_\pi K_1(m_\pi/T_{cr}) + 8\cosh(\mu_{cr}/T_{cr})\, m_N K_1(m_N/T_{cr})] \tag{15}$$

shown in fig. 2.3. The region denoted "Hadronic Gas Phase" is that described by our current approach.

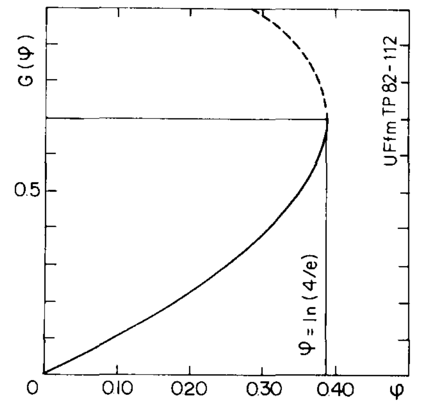

Fig. 2.2. Bootstrap function $G(\varphi)$ – the dashed line represents the unphysical branch.

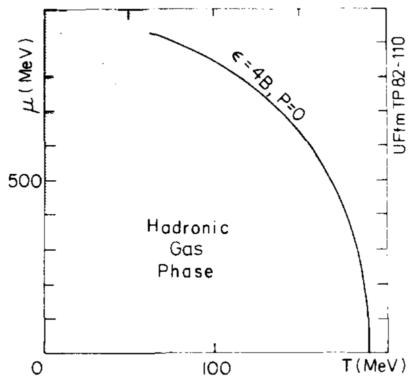

Fig. 2.3. Boundary to the "hadronic gas phase" in the bootstrap model. In the shaded region quantum statistics cannot be neglected.

With our choice of parameters we find that

$$T_{\rm cr}(\mu_{\rm cr}=0) = T_0 \sim 160\text{–}170\ \text{MeV}\,. \tag{16}$$

Note that $\mu = 0$ implies zero baryon number for the plasma state. For $\mu_0 = \mu_{\rm cr}$ ($T_{\rm cr} = 0$) the solution of eq. (15) is simply $\mu_{\rm cr} \sim m_{\rm N}$ since no quantum statistics effects have been included. Thus the dashed region in fig. 2.2 "nuclear matter" is excluded from our considerations. As we shall shortly see, the boundary of the hadronic gas phase is also characterized by a constant energy density $\varepsilon = 4B$.

Given the function $G(\varphi) = \phi(\beta, \lambda)$ we can in principle study the form of the hadronic mass spectrum. As it turns out we can obtain the partition function directly from ϕ: The formal similarity between eq. (3) and eq. (12) can be exploited to derive a relation between their integral transforms [1] (from here on: $\beta = \sqrt{\beta_\mu \beta^\mu}$);

$$\ln Z(\beta, V_{\rm ex}, \lambda) = -\frac{2\Delta(V_{\rm ex})}{\cancel{H\ (2\pi)^3}}\frac{\partial}{\partial\beta}\phi(\beta, \lambda) \tag{17}$$

which can also be written in a form which makes the different physical inputs more explicit:

$$\ln Z(\beta, V_{\rm ex}, \lambda) = \frac{\Delta(V_{\rm ex})}{\cancel{V_{\rm ex}}}\cdot\frac{\partial G(\varphi)}{\cancel{\partial\varphi}}\cdot Z_1(\beta, \lambda, V)\,. \tag{18}$$

In the absence of a finite hadronic volume and with interactions described by the first two terms, we would simply have an ideal Boltzmann gas, described by the one-particle partition function Z_1:

$$Z_1 = Z_{\rm q\bar q} + 2\cosh(\mu/T)\, Z_{\rm qqq} \tag{19}$$

where

$$Z_{\rm q\bar q}/Z_{\rm qqq} \approx (2I+1)(2S+1)\frac{VT^3}{2\pi^2}\left(\frac{m_{\pi/\rm N}}{T}\right)^2 K_2\left(\frac{m_{\pi/\rm N}}{T}\right). \tag{20}$$

Let us now briefly discuss the rôle of the available volume: as we have explicitly assumed, all hadrons have an internal energy density $4B$ (actually at finite pressure there is a small correction, see ref. [4a] for details). Hence the total energy of the fireball $E_{\rm F}$ can be written as

$$E_{\rm F} \equiv \varepsilon V_{\rm ex} = 4B(V_{\rm ex} - \Delta) \tag{21}$$

where $V_{\rm ex} - \Delta$ is the volume occupied by hadrons. We thus find

$$\Delta = V_{\rm ex} - E_{\rm F}/4B = V_{\rm ex}(1 - \varepsilon/(4B)) \tag{22}$$

when working out the relevant physical consequences we must always remember that the fireball is an isolated physical system, for which a statistical approach has been followed in view of the internal disorder (high number of available states) rather than because of a coupling to a heat bath.

The remainder of the discussion of the hadronic gas is a simple application of the rules of statistical thermodynamics. By investigating the meaning of the thermodynamic averages it turns out that the apparent (β, λ) dependence of the available volume Δ in eq. (18) must be disregarded when differentiating $\ln Z$ with respect to β and λ. As eq. (1) shows explicitly, the density of states for extended particles in V_{ex} is the same as that for point particles in Δ. Therefore

$$\ln Z(\beta, V_{ex}, \lambda) \equiv \ln Z_{pt}(\beta, \Delta, \lambda)\,. \tag{23}$$

We thus first calculate the *point* particle energy, baryon number densities, pressure, and entropy density

$$\varepsilon_{pt} = -\frac{1}{\Delta}\frac{\partial}{\partial\beta}\ln Z_{pt} = \frac{2}{H(2\pi)^3}\frac{\partial^2}{\partial\beta^2}\phi(\beta,\lambda) \tag{24}$$

$$\nu_{pt} = \frac{1}{\Delta}\lambda\frac{\partial}{\partial\lambda}\ln Z_{pt} = -\frac{2}{H(2\pi)^3}\lambda\frac{\partial^2}{\partial\lambda\,\partial\beta}\phi(\beta,\lambda) \tag{25}$$

$$P_{pt} = \frac{T}{\Delta}\ln Z_{pt} = -\frac{2T}{H(2\pi)^3}\frac{\partial}{\partial\beta}\phi(\beta,\lambda) \tag{26}$$

$$s_{pt} = \frac{1}{\Delta}\frac{\partial}{\partial T}(T\ln Z_{pt}) = \frac{P_{pt}}{T} + \frac{\varepsilon_{pt} - \mu\nu_{pt}}{T}\,. \tag{27}$$

From this, we easily find the energy density, as

$$\varepsilon = \frac{\langle E\rangle}{V_{ex}} = -\frac{1}{V_{ex}}\frac{\partial}{\partial\beta}\ln Z(\beta, V_{ex}, \lambda) = \frac{\Delta}{V_{ex}}\cdot\varepsilon_{pt}\,. \tag{28}$$

Inserting eq. (22) into eq. (28) and solving for ε, we find:

$$\varepsilon(\beta,\lambda) = \frac{\varepsilon_{pt}(\beta,\lambda)}{1 + \varepsilon_{pt}(\beta,\lambda)/4B}\,, \tag{29}$$

and hence another form for eq. (22):

$$V_{ex} = \Delta\cdot(1 + \varepsilon_{pt}(\beta,\lambda)/4B) \tag{30}$$

and similarly for the baryon density, pressure and entropy density

$$\nu = \frac{\nu_{pt}}{1 + \varepsilon_{pt}/4B} \tag{31}$$

$$p = \frac{P_{pt}}{1 + \varepsilon_{pt}/4B} \tag{32}$$

$$s = \frac{s_{pt}}{1 + \varepsilon_{pt}/4B}\,. \tag{33}$$

We now have a complete set of equations of state for observable quantities as functions of the chemical potential μ, the temperature T and the external volume V_{ex}. While these equations are semi-analytic, one has to evaluate the different quantities numerically due to the implicit definition of $\phi(\beta, \lambda)$ that determines $\ln Z$. However, when β, λ approach the critical curve, fig. 2.3, we easily find from the singularity of ϕ that ε_{pt} diverges and therefore

$$
\begin{aligned}
&\varepsilon \to 4B \\
&p \to 0 \\
&\Delta \to 0\,.
\end{aligned}
\tag{34}
$$

These limits indicate that at the critical line, matter has lumped into one large cluster with the energy density $4B$. No free volume is left, and, since only one cluster is present, the pressure has vanished. However, the baryon density varies along the critical curve; it falls with increasing temperature. This is easily understood: as temperature is increased, more mesons are produced that take up some of the available space. Therefore hadronic matter can saturate at lower baryon density. We further note here that in order to properly understand the approach to the phase boundary, one has to incorporate and understand the properties of the hadronic world beyond the critical curve. We turn now to the study of the perturbative quark-gluon plasma phase.

2.3. QCD and the quark-gluon plasma

We begin with a summary of the relevant postulates and results that characterize the current understanding of strong interactions in quantum chromodynamics (QCD). The most important postulate is that the proper vacuum state in QCD is not the (trivial) perturbative state that we (naively) imagine to exist everywhere and which is hardly changed when the interactions are turned off/on. In QCD the true vacuum state is believed to have a complicated structure which originates in the glue (pure gauge field) sector of the theory. The perturbative vacuum is an excited state with an energy density B above the true vacuum. It is to be found inside hadrons where perturbative quanta of the theory, in particular quarks, can therefore exist. The occurrence of the true vacuum state is intimately connected to the glue–glue interaction; gluons also carry the colour charge that is responsible for the quark–quark interaction. In the above discussion, the confinement of quarks is a natural feature of the hypothetical structure of the true vacuum.

Another feature of the true vacuum is that it exercises a pressure on the surface of the region of the perturbative vacuum to which quarks are confined. Indeed, this is just the idea of the original MIT bag model [10]. The Fermi pressure of almost massless light quarks is in equilibrium with the vacuum pressure B. When many quarks are combined to form a giant quark bag, then their properties inside can be obtained using the standard methods of many-body theory [2]. In particular, this also allows one to include the effect of internal excitation through a finite temperature and through a change in the chemical composition.

A further effect which must be taken into consideration is the quark–quark interaction. We shall use here the first order contribution in the QCD running coupling constant $\alpha_s(q^2) = g^2/4\pi$. However, as $\alpha_s(q^2)$ increases when the average momentum exchanged between quarks decreases, this approach will have only a limited validity at relatively low densities and/or temperatures. The collective screening effects in the plasma are of comparable order of magnitude and should reduce the importance of the perturbative contribution.

As u and d quarks are almost massless inside a bag, they can be produced in pairs and at moderate temperatures many $q\bar{q}$ pairs will be present. In particular also $s\bar{s}$ pairs will be produced and we will return to this point below. Furthermore, real gluons can be present when $T \neq 0$ and will be included here in our considerations.

As it was outlined in the previous section, a complete description of the thermodynamical behaviour of a many-particle system can be derived from the grand partition function Z. For the case of the quark-gluon plasma in the perturbative vacuum, one finds an analytic expression to first order in α neglecting quark masses. We obtain for the quark Fermi gas [2b]

$$\ln Z_q(\beta, \lambda) = \frac{gV}{6\pi^2}\beta^{-3}\left[\left(1-\frac{2\alpha_s}{\pi}\right)\left(\frac{1}{4}\ln^4\lambda_q + \frac{\pi^2}{2}\ln^2\lambda_q\right) + \left(1-\frac{50}{21}\frac{\alpha_s}{\pi}\right)\frac{7\pi^4}{60}\right] \tag{35}$$

where $g = (2s+1)(2I+1)N = 12$ counts the number of the components in the quark gas, and λ_q is the fugacity related to quark number. Since each quark has baryon number $\frac{1}{3}$, we find

$$\lambda_q^3 = \lambda = e^{\mu/T} \tag{36}$$

where λ, as previously, allows one to have conservation of baryon number. Consequently

$$3\mu_q = \mu\,. \tag{37}$$

The glue contribution is [2]

$$\ln Z_g(\beta, \lambda) = V\frac{8\pi^2}{45}\beta^{-3}\left(1-\frac{15\alpha_s}{4\pi}\right). \tag{38}$$

We notice two relevant differences with the photon gas: (i) the occurrence of a factor eight associated with the number of gluons; (ii) the glue–glue interaction since gluons carry the colour charge.

Finally, let us introduce the true vacuum term as

$$\ln Z_{vac} = -\beta BV\,. \tag{39}$$

This leads to the required positive energy density B within the volume occupied by the coloured quarks and gluons and to a negative pressure on the surface of this region. At this stage, this term is entirely phenomenological as discussed above. The equations of state for the quark-gluon plasma are easily obtained by differentiating

$$\ln Z = \ln Z_q + \ln Z_g + \ln Z_{vac} \tag{40}$$

with respect to β, λ and V. The energy density, baryon number density, pressure and entropy density are respectively, written in terms of μ and T

$$\varepsilon = \frac{6}{\pi^2}\left[\left(1-\frac{2\alpha_s}{\pi}\right)\left(\frac{1}{4}\left(\frac{\mu}{3}\right)^4 + \frac{1}{2}\left(\frac{\mu}{3}\right)^2(\pi T)^2\right) + \left(1-\frac{50}{21}\frac{\alpha_s}{\pi}\right)\frac{7}{60}(\pi T)^4\right] + \frac{8}{15\pi^2}(\pi T)^4\left(1-\frac{15}{4}\frac{\alpha_s}{\pi}\right) + B \tag{41}$$

$$\nu = \frac{2}{3\pi^2}\left[\left(1 - \frac{2\alpha_s}{\pi}\right)\left(\left(\frac{\mu}{3}\right)^3 + \frac{\mu}{3}(\pi T)^2\right)\right] \tag{42}$$

$$p = \frac{1}{3}(\varepsilon - 4B) \tag{43}$$

$$s = \frac{2}{\pi}\left(1 - \frac{2\alpha_s}{\pi}\right)\left(\frac{\mu}{3}\right)^2(\mu T) + \frac{14}{15\pi}\left(1 - \frac{50}{21}\frac{\alpha_s}{\pi}\right)(\pi T)^3 + \frac{32}{45\pi}\left(1 - \frac{15}{4}\frac{\alpha_s}{\pi}\right)(\pi T)^3 . \tag{44}$$

In eqs. (41, 44) the second T^4 (respt. T^3) term originates from the gluonic degrees of freedom. In eq. (43) we have right away used the relativistic relation between the quark and gluon energy density and pressure

$$p_q = \tfrac{1}{3}\varepsilon_q , \qquad p_g = \tfrac{1}{3}\varepsilon_g \tag{45}$$

in order to derive this simple form of the equation of state.

This simple equation of state of the quark-gluon plasma is slightly modified when finite quark masses are considered, or when the QCD coupling constant α_s is dependent on the dimensional parameter Λ. From eq. (43) it follows that when the pressure vanishes, the energy density is $4B$, independently of the values of μ and T which fix the line $P = 0$. We recall that this has been precisely the kind of behaviour found for the hadronic gas. This coincidence of the physical observables strongly suggests that, in an exact calculation, both lines $P = 0$ should coincide; we will return to this point again below. For $P > 0$ we have $\varepsilon > 4B$ – we recall that in the hadronic gas we always had $\varepsilon \leq 4B$. Thus, in this domain of the μ–T plane, we have a quark-gluon plasma exposed to an external force.

In order to obtain an idea of the form of the ($P = 0$) critical curve in the μ–T plane as obtained for the quark-gluon plasma, we rewrite eq. (43) for $P = 0$:

$$B = \frac{(1 - 2\alpha_s/\pi)}{162\pi^2}[\mu^2 + (3\pi T)^2]^2 - \frac{T^4\pi^2}{45}\left[\left(1 - \frac{5\alpha_s}{3\pi}\right)\cdot 12 - \left(1 - \frac{15\alpha_s}{4\pi}\right)\cdot 8\right]. \tag{46}$$

Here, the last term is the glue pressure contribution. We find that the greatest lower bound on temperature T_q, at $\mu = 0$ is about ($\alpha_s = \frac{1}{2}$)

$$T_q \approx 0.83\, B^{1/2} \sim 160\ \mathrm{MeV} \approx T_0 . \tag{47}$$

This result shows the expected order of magnitude. The most remarkable point is, that it leads, for $B^{1/4} = 190$ MeV, to almost exactly the same value as that found in the hadronic gas study presented in the previous section.

Let us further note here that for $T \ll \mu$ the baryon chemical potential tends to

$$\mu_B = 3\mu_q \Rightarrow 3B^{1/4}\left[\frac{2\pi^2}{(1 - 2\alpha_s/\pi)}\right]^{1/4} = 1320\ \mathrm{MeV} \quad [\alpha_s = \tfrac{1}{2},\ B^{1/4} = 190\ \mathrm{MeV}] . \tag{48}$$

Concluding this discussion of the $P = 0$ line for the quark-gluon plasma, let us note that the choice $\alpha_s \sim \frac{1}{2}$ is motivated by fits of the charmonium and upsilonium spectra as well as by the analysis of deep inelastic scattering. In both these cases spacelike domains of momentum transfer are explored. The much smaller value of $\alpha_s \sim 0.2$ is found in timelike regions of momentum transfer, in $e^+e^- \to$ hadrons experiments. In the quark-gluon plasma, as described up to first order in perturbation theory, positive and negative momentum transfers occur: the perturbative corrections to the radiative T^4 contribution is dominated by timelike momentum transfers, while the correction to the μ^4 term originates from spacelike quark–quark scattering. Finally we consider the energy density at $\mu = 0$. Restructuring some factors again, we find the simple result:

$$\varepsilon = B + \frac{\pi^2}{30} T^4 \left[2_s \cdot 8_c \cdot \left(1 - \frac{15}{4}\frac{\alpha_s}{\pi}\right) + 2_I \cdot 2_s \cdot 2_c \frac{7}{4}\left(1 - \frac{50}{21}\frac{\alpha_s}{\pi}\right)\right] . \tag{49}$$

We note that for both quarks and gluons the interaction conspires to *reduce* the effective number of degrees of freedom which are accessible. At $\alpha_s = 0$ we find a handy relation

$$\varepsilon_q + \varepsilon_g = (T/160\,\mathrm{MeV})^4[\mathrm{GeV/fm}^3] . \tag{50}$$

At $\alpha_s = \frac{1}{2}$ we are seemingly left with only ~50% of the degrees of freedom, and the temperature "unit" in the above formula drops to 135 MeV.

I have so far neglected to include heavy flavours into the description. For charm, with a mass of about 1500 MeV, the thermodynamic abundance is sufficiently low that we can ignore its influence on the properties of the plasma. Also, even the equilibrium abundance is quite small. Evaluating the phase-space integrals that the ratio of charm to light antiflavour (either $\bar{u}$ or $\bar{d}$) gives

$$c/\bar{q} = \bar{c}/\bar{q} = \exp\{-(m_c - \mu/3)/T\}(m_c/T)^{3/2}\tfrac{1}{2}\sqrt{\pi/2} . \tag{51}$$

Taking as a numerical example $m_c = 1500$ MeV, $T = 200$ MeV, $\mu = 0$, one finds with $c/\bar{q} = 7 \times 10^{-3}$ a small, but still quite significant abundance. However, the approach to chemical equilibrium (see below) is to be studied to establish if the chemical equilibrium assumption is justified. We note that the energy fraction carried by intrinsic charm in the plasma would be ~0.2% in the above example.

Clearly, we must turn our attention to strangeness – with a current quark mass of about 180 MeV, we are actually above threshhold and indeed one finds that there is a quite appreciable s-abundance (see again next part). An explicit calculation [4b] has shown that chemical equilibrium will be reached during the short time interval of a heavy ion reaction. The motion of the particles being already semirelativistic, an increase by about 15% of the number of available degrees of freedom (eq. (49)) is due to $s\bar{s}$ production. The appearance of strangeness is a very important qualitative factor and we shall return to its discussion in section 2.5.

2.4. Phase transition from the hadronic gas to the quark-gluon plasma

We have shown that two inherently different descriptions lead to the prediction of a qualitatively similar region where a transition between both phases of hadronic matter can occur. From our results we cannot deduce the order of the phase transition. However, the physics arguments which went into these theoretical approaches require that this is a first order phase transition.

Consider the p–V diagram shown in fig. 2.4. Here we distinguish three domains – the hadonic gas region is simply a Boltzmann gas where the pressure increases with reduction of the volume. However,

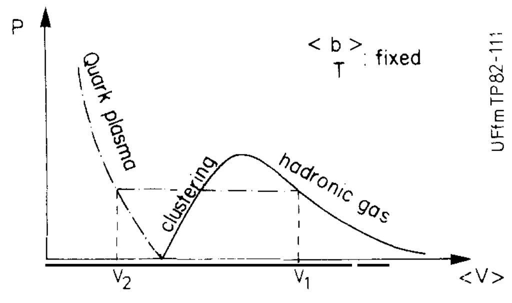

Fig. 2.4. p–V diagram for the gas-plasma first order transition.

when internal excitation becomes important, the individual hadrons begin to cluster, reducing the increase in the Boltzmann pressure since smaller number of particles exercises smaller pressure. In the proper description we would have to describe this situation by allowing a coexistence of hadrons with the plasma – this becomes necessary when the clustering overwhelms the compressive effects and the pressure falls to zero as V reaches the proper volume of hadronic matter. At this point the pressure rises again very quickly, since we now compress the hadronic constituents. By performing the Maxwell construction as indicated in fig. 2.4 between volumes V_1 and V_2 we can find the most likely way taken by the compressed hadronic gas in a nuclear collision. In our approach it seems to be a first order transition. We should remember, that on the way out, during the expansion of the plasma state, the entropy generated in the plasma (e.g. by s-production, shocks etc.) may require that the isolated plasma state must expand to vanishing pressure $P = 0$ before it can disintegrate into individual hadrons. In an extreme situation this disintegration may be quite a slow process with successive fragmentations!

It is interesting to follow the path taken by an isolated quark-gluon plasma fireball in the μ–T plane, or equivalently in the ν–T plane. Several cases are depicted in fig. 2.5. After the Big Bang, with expansion of the universe, the cooling shown by the dashed line occurs in a universe in which most of the energy is in the form of radiation – hence we have for the chemical potential $\mu \ll T$. Similarly the baryon density ν is quite small. In normal stellar collapse leading to cold neutron stars we follow the dashed-dotted line parallel to the μ- resp. ν-axis. The compression is accompanied by little heating. In nuclear collision shown by the full line, the entire μ–T and ν–T plane can be explored by varying the parameters of the colliding nuclei. It is important to appreciate that the arrows show the time evolution, i.e. path of increasing entropy for the hadronic fireball at fixed total energy and baryon number.

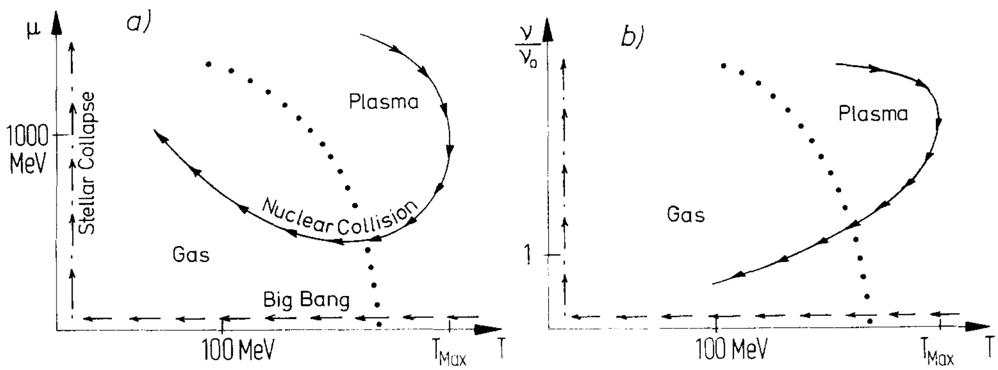

Fig. 2.5. Paths taken in the (a) μ–T plane and (b) ν–T plane by different physical events.

In the expansion period during which the temperature decreases, there is an associated decrease of the chemical potential and of the density in the plasma phase while in the hadronic *gas* phase the chemical potential can increase while the baryon density decreases. As it is evident from fig. 2.5, one expects that the transition *from gas to plasma* takes place at higher baryon density and lower temperature than the transition *from plasma to gas.* Obviously the larger volume fireball at higher temperature contains more entropy at fixed total energy and baryon number. The initial heating of the fireball at almost constant baryon density is done at the expense of a significant reduction in the baryon chemical potential. This conversion of chemical energy to thermal excitation stops at some T_{Max} the value of which depends on the available internal fireball energy. The qualitative curves are typical representatives obtained from the equations of sections 2.2 and 2.3 for fixed E, b. Finally, the question arises: how does the hadronic gas enter into the plasma state? As we follow the full line backwards, μ (resp. ν) increases with decreasing T and we stay in the plasma phase until quite low temperatures. This suggests that in order to get into the plasma at moderate temperatures and baryon densities (say: $T = 150$ MeV, $\nu \sim 3\nu_0 \leftrightarrow \mu \sim 800$ MeV) we must blow off (perhaps in a manner similar to supernovae explosions) some cold surface matter – or otherwise generate by internal nonequilibirum processes *sufficient amounts of entropy.* It is for that reason that we have avoided to indicate the gas → plasma transition in fig. 2.5, as it must be a highly nonequilibrium transition to which values μ, T cannot perhaps be assigned at all: On the other hand, the expansion of the plasma seems to be an adiabatic process, although here also some significant amounts of entropy are produced.

As a last related comment we turn to the question: is the transition "hadronic gas → quark-gluon plasma" in principle a phase transition or is it only a change in the nature of hadronic matter which is not associated with any kind of singularity in the partition function in the limit of infinite volume. In the spirit of the theoretical approaches taken here one needs a first order transition. However, this cannot be considered as final – since contrary evidence can be found arguing that, in any finite volume, only a finite number of *incompressible* hadrons can be studied. Here it turns out that one must very carefully study the meaning of the thermodynamical limits before a conclusion can be reached; even worse is the observation that for compressible individual hadrons we might find a second order phase transition. From this remark we learn how sensitive this theory is to even the slightest improvement. I would like to conclude that it is experiment which should teach us this important aspect of strong interactions.

2.5. Strangeness in the plasma

In order to observe the properties of the quark-gluon plasma we must design a thermometer, or an isolated degree of freedom weakly coupled to hadronic matter. Nature has provided several such thermometers: leptons, direct photons and quarks of heavy flavours. We would like to point here to a particular phenomenon perhaps quite uniquely characteristic of quark matter. First we note that, at a given temperature, the quark-gluon plasma will contain an equal number of strange (s) quarks and antistrange ($\bar{s}$) quarks. They are present during a hadronic collision time much too short to allow for weak interaction conversion of light flavours to strangeness. Assuming chemical equilibrium in the quark plasma, we find the density of the strange quarks to be (two spins and three colour):

$$\frac{s}{V} = \frac{\bar{s}}{V} = 6 \int \frac{d^3p}{(2\pi)^3} \exp\{-\sqrt{p^2 + m_s^2}/T\} = 3 \frac{T m_s^2}{\pi^2} K_2\left(\frac{m_s}{T}\right) \tag{52}$$

(neglecting, for the time being, perturbative corrections). As the mass of the strange quarks, m_s, in the perturbative vacuum is believed to be of the order of 150–280 MeV, the assumption of equilibrium for $m_s/T \sim 2$ may indeed be correct. In eq. (52) the Boltzmann distribution can be used, as the density of strangeness is relatively low. Similarly, there is a certain light antiquark density ($\bar{q}$ stands for either $\bar{u}$ or $\bar{d}$):

$$\frac{\bar{q}}{V} \cong 6 \int \frac{d^3p}{(2\pi)^3} \exp\{-|p|/T - \mu_q/T\} = \exp\{-\mu_q/T\} \cdot T^3 \frac{6}{\pi^2} \tag{53}$$

where the quark chemical potential is $\mu_q = \mu/3$. This exponent suppresses the $q\bar{q}$ pair production, since only for energies higher than μ_q, there is a large number of empty states available for quarks.

What I now intend to show is that there are many more $\bar{s}$ quarks than antiquarks of each light flavour. Indeed:

$$\frac{\bar{s}}{\bar{q}} = \frac{1}{2} \left(\frac{m_s}{T}\right)^2 K_2\left(\frac{m_s}{T}\right) e^{\mu/3T} . \tag{54}$$

The function $x^2 K^2(x)$ varies between 1.3 and for $x = m_s/T$ between 1.5 and 2. Thus, we almost always have more $\bar{s}$ than $\bar{q}$ quarks and, in many cases of interest, $\bar{s}/\bar{q} \sim 5$. As $\mu \to 0$ there are about as many $\bar{u}$ and $\bar{q}$ quarks as there are $\bar{s}$ quarks. This is shown quantitatively in fig. 2.6. Another important aspect is the total strangeness abundance since for $T = 200$ MeV, $m_s = 150$ MeV, chemical equilibrium predicts it at about twice the normal baryon density: $s/b = 0.4$; hence there are as many strange and antistrange quarks as there are baryons in the hadronic gas, or even much more, if we are in the "radiation" i.e. baryon number depleted region.

The crucial question which arises is whether there is enough time to create $s\bar{s}$ pairs in nuclear collisions. To answer it one has to compute [4b] (say in lowest order in perturbative QCD) the two contributing invariant reaction rates (per unit time and per unit volume)

$$\begin{aligned} &A_{q\bar{q}} : q\bar{q} \to s\bar{s} \\ &A_{gg} : gg \to s\bar{s} . \end{aligned} \tag{55}$$

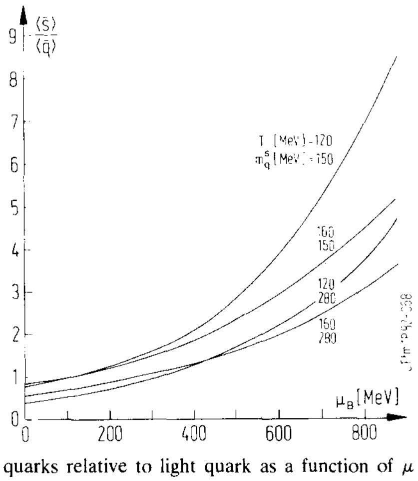

Fig. 2.6. Abundance of strange (= antistrange) quarks relative to light quark as a function of μ for several choices of T (= 120, 160 MeV) and strange quark mass (m_q^s = 150, 280 MeV).

The contributing diagrams are shown in figs. 2.7 a and b, respectively. These rates are dominated by the glue–glue reaction and at $T = 200$ MeV, $m_s = 150$ MeV, $\alpha_s = 0.6$ one finds $A_{gg} \approx 16/\mathrm{fm}^4$. This is quite a large rate, indicating that the typical relaxation time

$$\tau = n(\infty)/A \tag{56}$$

($n(\infty)$ is the density at infinite time) will be about 10^{-23} sec. In fig. 2.8 the strangeness population evolution is shown as a function of time at fixed $\mu = 900$ MeV. During the minimal anticipated lifetime of the plasma we thus find that the strange quark abundance saturates at its chemical equilibrium point.

One can study how much more total strangeness is found in the quark-gluon plasma as compared to the hadronic gas phase. While the total yields are up to 5–7 times higher (again depending on some parameters) it is more appropriate to concentrate attention on those reaction channels which will be particularly strongly populated when the quark plasma dissociates into hadrons. Here in particular, it appears that the presence of quite rare multistrange hadrons will be enhanced, first because of the relative high phase space density of strangeness in the plasma, and second because of the attractive ss-QCD interaction in the $\bar{3}_c$ state and $\bar{s}s$ in the 1_c state. Hence one should search for an increase of the abundances of particles like Ξ, $\bar{\Xi}$, Ω, $\bar{\Omega}$, ϕ and perhaps for highly strange pieces of baryonic matter, rather than in the K-channels. However, it appears that already a large value for the $\bar{\Lambda}/\Lambda$ ratio would be a significant signal. Not to be forgotten are secondary effects, e.g. those due to $s\bar{s}$ annihilation into γ (and infrared glue) in the plasma. Different experiments will be sensitive to different energy ranges.

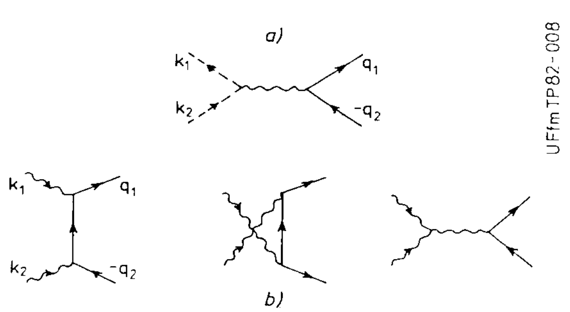

Fig. 2.7. First order diagrams for $s\bar{s}$ production reactions; (a) $q\bar{q} \to s\bar{s}$, (b) $gg \to s\bar{s}$.

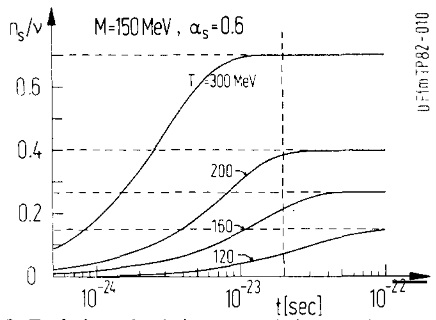

Fig. 2.8. Evolution of relative s population per baryon number as function of time in the plasma. For $T \gtrsim 160$ MeV chemical saturation is noticeable in about 2×10^{-23} sec, the anticipated minimal plasma livetime.

2.6. Summary and outlook

Our aim has been to obtain a description of highly excited hadronic matter. By postulating a kinetic and chemical equilibrium we have been able to develop a thermodynamic description valid for high temperatures and different chemical compositions. Along this line we have found two physically different domains; firstly a hadronic gas phase, in which individual hadrons can exist as separate entities, but are sometimes combined into larger hadronic clusters; and secondly, a domain in which individual hadrons dissolve into one large cluster consisting of hadronic constituents – the quark-gluon plasma.

In order to obtain a theoretical description of both phases we have used some "common" knowledge and a plausible interpretation of the currently available experimental facts. In particular, in the case of the hadronic gas, we have completely abandoned a more conventional Lagrangian approach in favour of a semiphenomenological statistical bootstrap model of hadronic matter that incorporates those properties of hadronic interaction which are, in our opinion, most important.

In particular, the attractive interactions are included through the rich, exponentially growing hadronic mass spectrum $\tau(m^2, b)$ while the introduction of a finite volume for each hadron is responsible for an effective short-range repulsion. We have neglected quantum statistics in the hadronic gas phase since a quantitative study reveals that this is allowed above $T \approx 50$ MeV. But we allow particle production, which introduces a quantum physical aspect into the otherwise "classical" theory of Boltzmann particles.

Our considerations lead us to an equation of state for hadronic matter which reflects what we have included in our considerations. It is the *quantitative* nature of this approach that allows a detailed comparison with experiment. It is important to observe that the predicted temperatures and mean transverse momenta of particles agree with the experimental results available at $E_{\mathrm{k,lab}}/A = 2$ GeV [BEVELAC –] and at 100 GeV [ISR –] as much as a comparison is permitted.

The internal theoretical consistency of this description of the gas phase leads, in a straightforward fashion, to the postulate of a first order phase transition to a quark-gluon plasma. This second phase is treated by a quite different method; in addition to the standard Lagrangian quantum field theory of ("weakly") interacting particles at finite temperature and density, we also introduce the phenomenological vacuum pressure and energy density B. This term is required in a consistent theory of hadronic structure. It turns out that $B^{1/4} \sim 190$ MeV is just, to within 20%, the temperature of the quark phase before its dissociation into hadrons. This is similar to the maximal hadronic temperature $T_0 \simeq 160$ MeV.

Perhaps the most interesting aspect of our work is the realization that the transition to quark matter will occur at very much lower baryon density for highly excited hadronic matter than for matter in its ground state ($T = 0$). Using the currently accepted value for B, we find that at $\nu \sim 2$–$3\nu_0$, $T = 150$ MeV, a quark phase may indeed already be formed. The detailed study of the different aspects of this phase transition must still be carried out. However, initial results look very encouraging, since the required baryon density and temperatures are well within the range of fixed target, heavy nucleon collisions with 100 GeV per nucleon. We look forward to such a heavy ion facility which should provide us with the required experimental information.

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