Quark-Hadron Universe

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REPORTING WORK CARRIED OUT WITH

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- **[I] Introduction, hadronization time constant**
- **[II] Chemical potentials and evolution constraints**
- **[III] Particle abundances**
- **[IV] Distillation process**
- **[V] Baryon-Antibaryon symmetry (CPT)**

QUESTIONS: baryon asymmetry, cosmic strangeletts, N/P ratio

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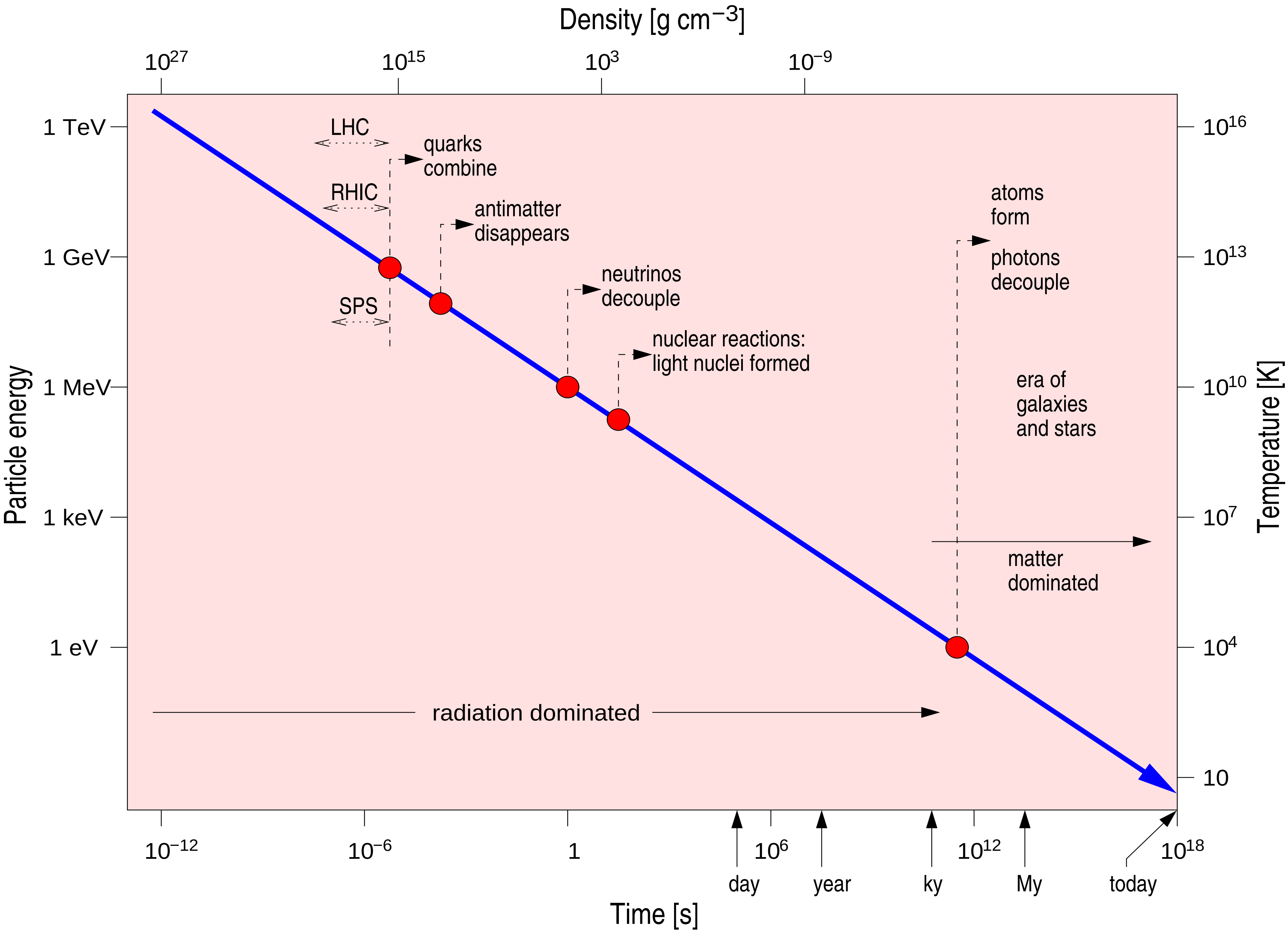
Some reprints available.

Short History of the Universe



Quark-Hadron Universe

Our objective here is to study the Universe in the range $300 < T < 5$ MeV in which the nearly free gas of quarks and gluons hadronizes and the antimatter rich Universe annihilates into the final particle content as seen today.

Quantitative Tasks

1) Obtain the chemical potentials in the early Universe as function of time;

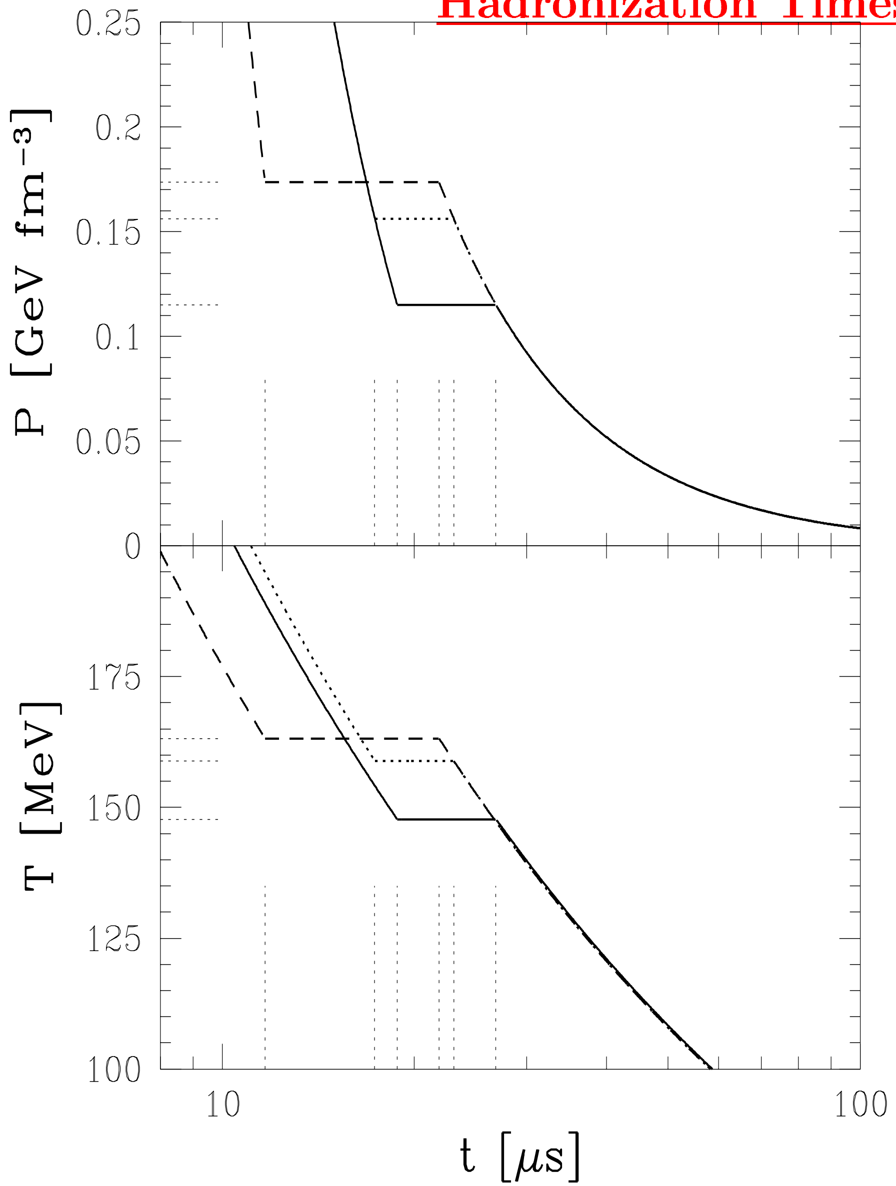
2) Understand the chemical (particle) composition of the Universe;

3) Explore the quark-hadron phase transformation dynamics, and establish potential for baryon number distillation;

4) Describe the flavor composition of the Universe during evolution toward the condition of neutrino decoupling at

$$T \simeq 1\,\textbf{MeV} \qquad t \simeq 10\,s$$

Hadronization Timescale in the Universe



Two dynamical equations:

Entropy conserving expansion:

$$dE + P\,dV = T\,dS = 0, \quad dE = d(\epsilon V),$$

$$\frac{dV}{V} = \frac{3\,dR}{R}, \quad \frac{3dR}{R} = -\frac{d\epsilon}{\epsilon + P}.$$

Contraction of the Einstein equation in Freedman coordinates (Robertson-Walker Universe):

$$\mathcal{R}_{\mu\nu} - \frac{1}{2} g_{\mu\nu}\mathcal{R} + \Lambda_{\rm v} g_{\mu\nu} = 8\pi G T_{\mu\nu},$$

$$\epsilon = 3P + 4\mathcal{B}, \quad \dot{\epsilon}^2 = \frac{128\pi G}{3}\,\epsilon\,(\epsilon - \mathcal{B})^2,$$

Solution:

$$\epsilon_1 = \mathcal{B}\coth^2(t/\tau_{\rm U}),$$

TIME CONSTANT:

$$\tau_{\rm U} = \sqrt{\frac{3c^2}{32\pi G\mathcal{B}}} = 36\sqrt{\frac{\mathcal{B}_0}{\mathcal{B}}}\,\mu\mathbf{s}, \quad \mathcal{B}_0 = 0.19\,\frac{\mathbf{GeV}}{\mathbf{fm}^3}$$

Pressure (upper) and temperature (lower part) in the Universe, as function of time, in the vicinity of the phase transition from the deconfined phase to the confined phase. Solid lines, $\mathcal{B}^{1/4} = 195$ MeV; dotted lines, $\mathcal{B}^{1/4} = 170$ MeV (lower part) and $\mathcal{B}^{1/4} = 220$ MeV (upper part) all for $\alpha_s = 0.6$.

CHEMICAL POTENTIALS IN THE UNIVERSE

The time constant of hadronization implies equilibrium is reached for all hadronic reactions (phase space occupancy saturated), and there is full participation of photon and lepton degrees of freedom.

- **Photons in chemical equilibrium, assume the Planck distribution, implying a zero photon chemical potential; i.e., $\mu_\gamma = 0$.**
- **Because reactions such as $f + \bar{f} \rightleftharpoons 2\gamma$ are allowed, where f and $\bar{f}$ are a fermion – antifermion pair, we immediately see that $\mu_f = -\mu_{\bar{f}}$ whenever chemical and thermal equilibrium have been attained.**
- **More generally for any reaction $\nu_i A_i = 0$, where ν_i are the reaction equation coefficients of the chemical species A_i, chemical equilibrium occurs when $\nu_i \mu_i = 0$, which follows from a minimization of the Gibbs free energy.**
- **Weak interaction reactions assure:**

$$\mu_e - \mu_{\nu_e} = \mu_\mu - \mu_{\nu_\mu} = \mu_\tau - \mu_{\nu_\tau} \equiv \Delta\mu_l, \qquad \mu_u = \mu_d - \Delta\mu_l, \qquad \mu_s = \mu_d\ ,$$

- **For the experimentally-favored "large mixing angle" solution the neutrino oscillations $\nu_e \rightleftharpoons \nu_\mu \rightleftharpoons \nu_\tau$ imply that:**

$$\mu_{\nu_e} = \mu_{\nu_\mu} = \mu_{\nu_\tau} \equiv \mu_\nu,$$

note that the mixing is occurring fast in 'dense' matter.

- **There are three chemical potentials which are 'free' and we choose to follow the following: μ_d, μ_e, and μ_ν.**

- **Quark chemical potentials characterize the particle abundances in the hadron phase, e.g. $\Sigma^0\,(uds)$ has chemical potential $\mu_{\Sigma^0} = \mu_u + \mu_d + \mu_s$**

- **The baryochemical potential is:**

$$\mu_b = \frac{\mu_P + \mu_N}{2} = \frac{1}{3}\,\frac{\mu_d + \mu_u}{2} = 3\mu_d - \frac{3}{2}\Delta\mu_l = 3\mu_d - \frac{3}{2}(\mu_e - \mu_\nu).$$

Chemical Conditions

The three chemical potentials are obtained solving the three constraints:

i. *Charge neutrality* **($Q = 0$) is required to eliminate Coulomb energy. This implies that:**

$$n_Q \equiv \sum_i Q_i\, n_i(\mu_i, T) = 0,$$

where Q_i and n_i are the charge and number density of species i.

ii. *Net lepton number equals net baryon number* **($L = B$) is the most elegant baryogenesis model:**

$$n_L - n_B \equiv \sum_i (L_i - B_i)\, n_i(\mu_i, T) = 0,$$

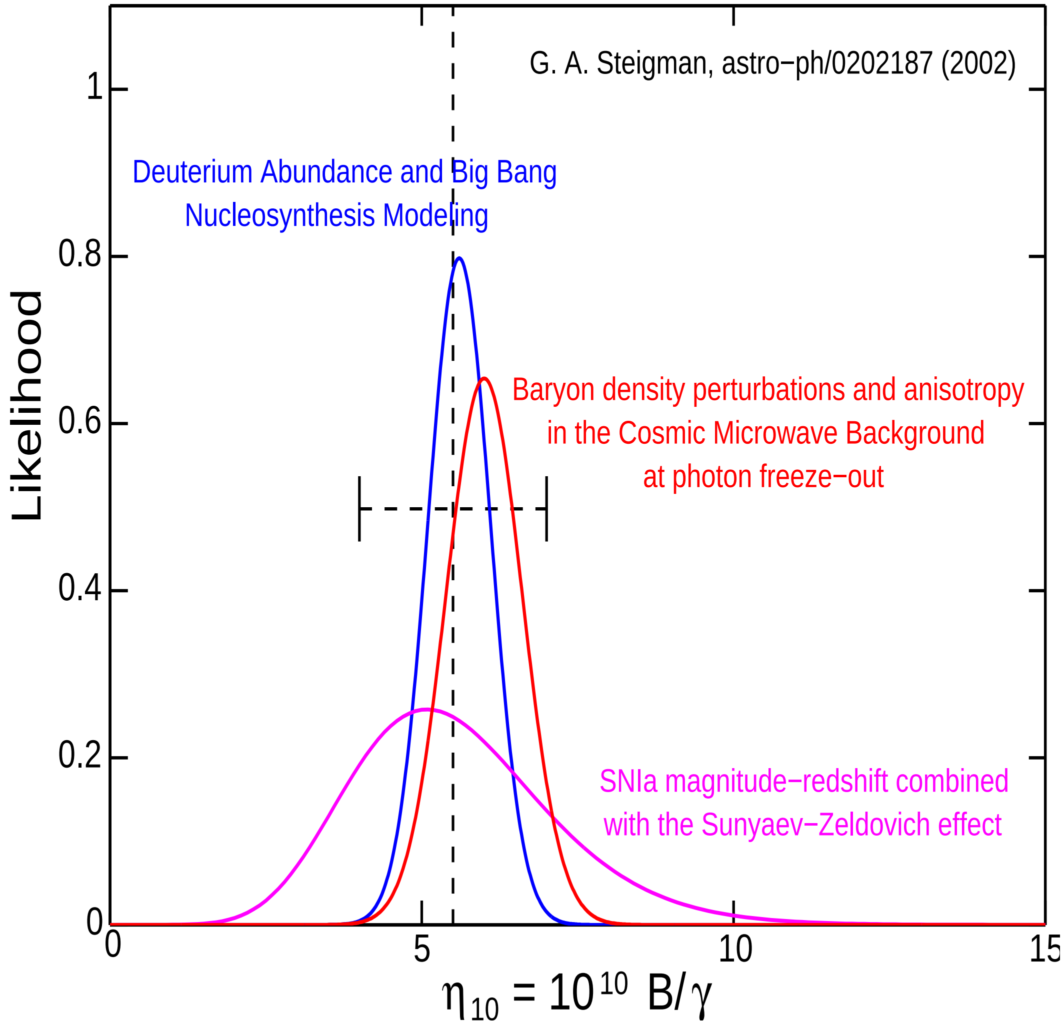
(generalization to finite $B - L$ easily possible if desired, has no impact)

iii. Universe evolves adiabatically i.e. at *constant in time entropy-per-baryon* **S/B the ,**

$$\frac{\sigma}{n_B} \equiv \frac{\sum_i \sigma_i(\mu_i, T)}{\sum_i B_i\, n_i(\mu_i, T)} = 4.5^{+1.4}_{-1.1} \times 10^{10} \quad \Longrightarrow$$

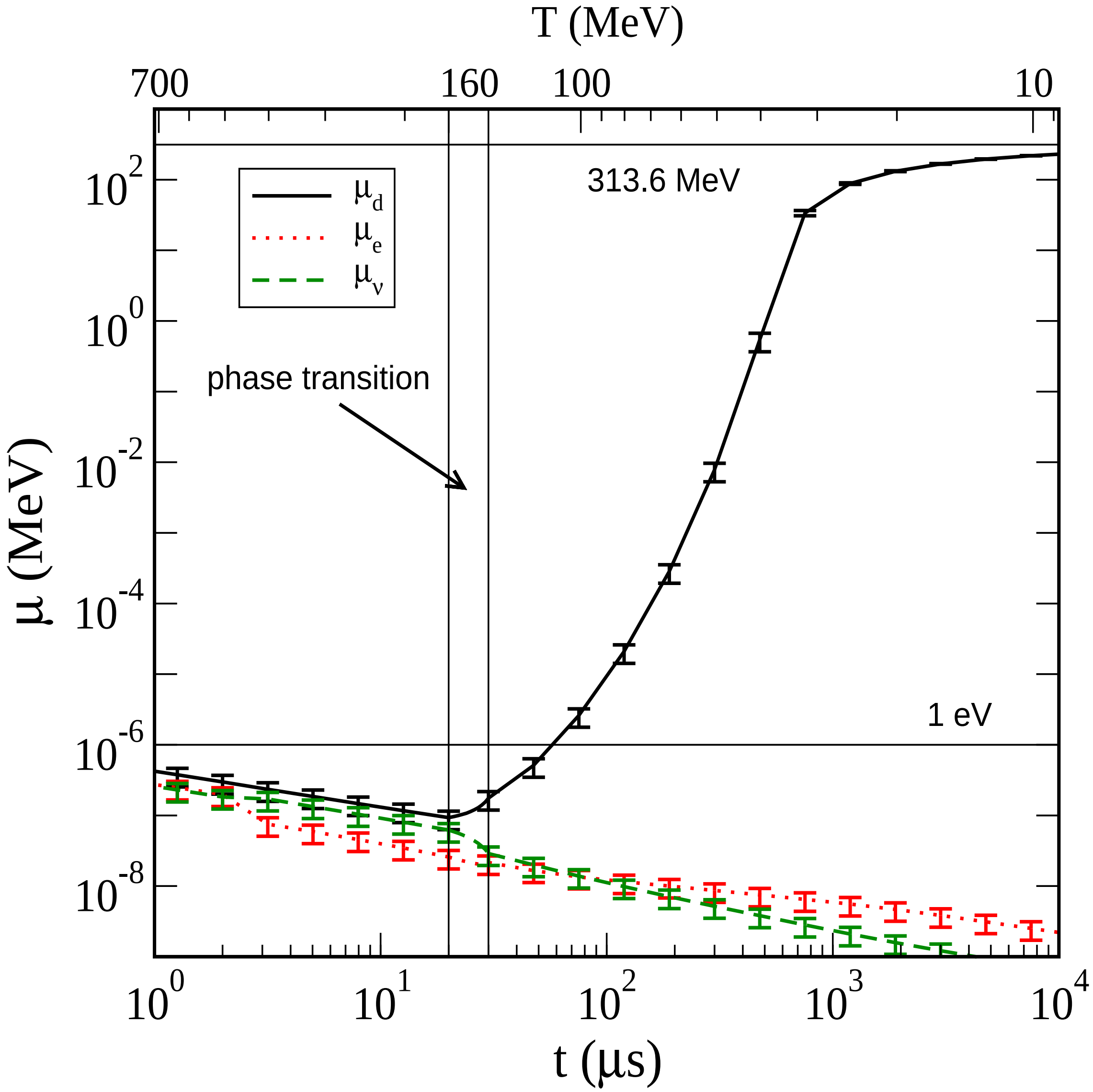
Entropy per Baryon in the Universe

$$\eta \equiv n_{\rm B}/n_\gamma = 5.5 \pm 1.5 \times 10^{-10}$$

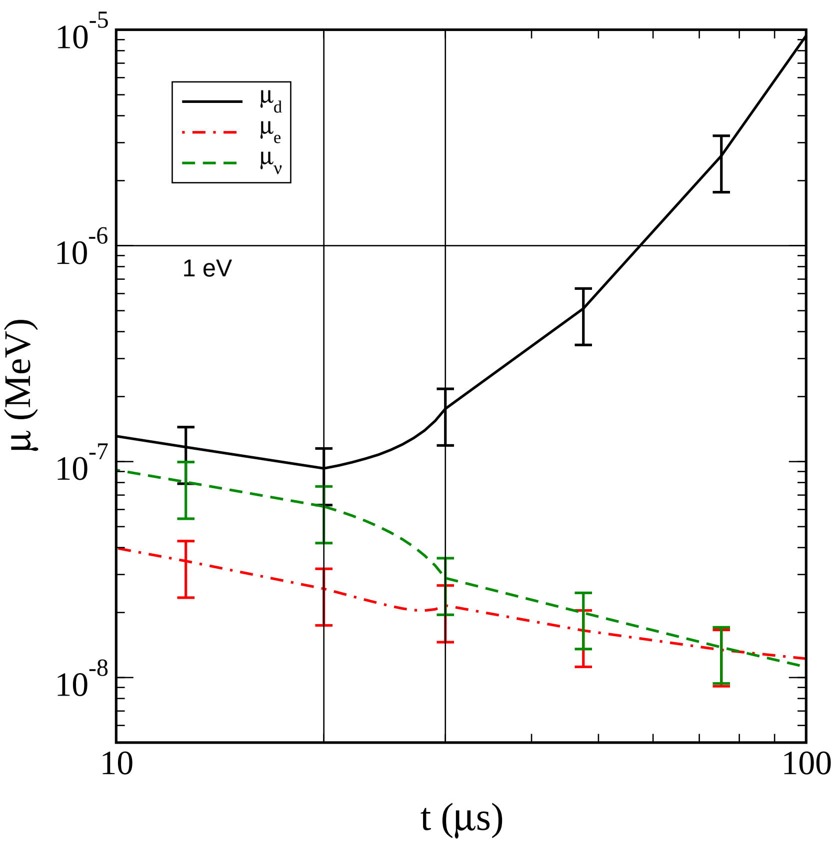


This yields $S/b \simeq 4.5\,10^{10}$

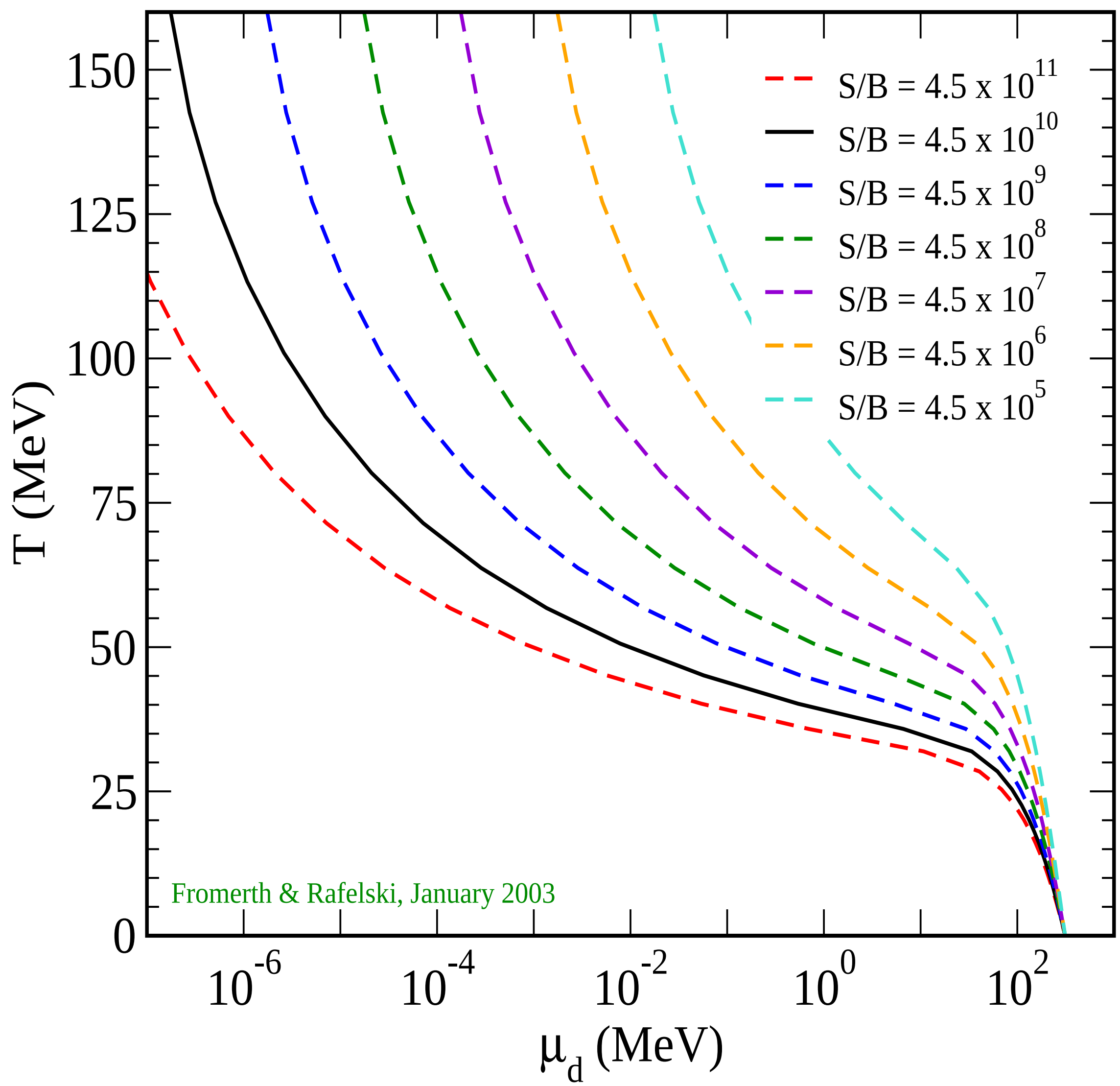
TRACING μ_d IN THE UNIVERSE



Minimum $\mu_b = 0.33^{+0.11}_{-0.08}$ eV.



TRACING μ_d IN A UNIVERSE



Mixed Phase

Many properties of the Universe non-continuous comparing pure QGP with Hadron Phase. We need mixed hadron-quark phase: we parameterize the partition function during the phase transformation as

$$\ln Z_{\rm tot} = f_{\rm HG} \ln Z_{\rm HG} + (1 - f_{\rm HG}) \ln Z_{\rm QGP}$$

$f_{\rm HG}$ **represents the fraction of total phase space belonging to the HG phase.**

The three constraints are accordingly modified, e.g.:

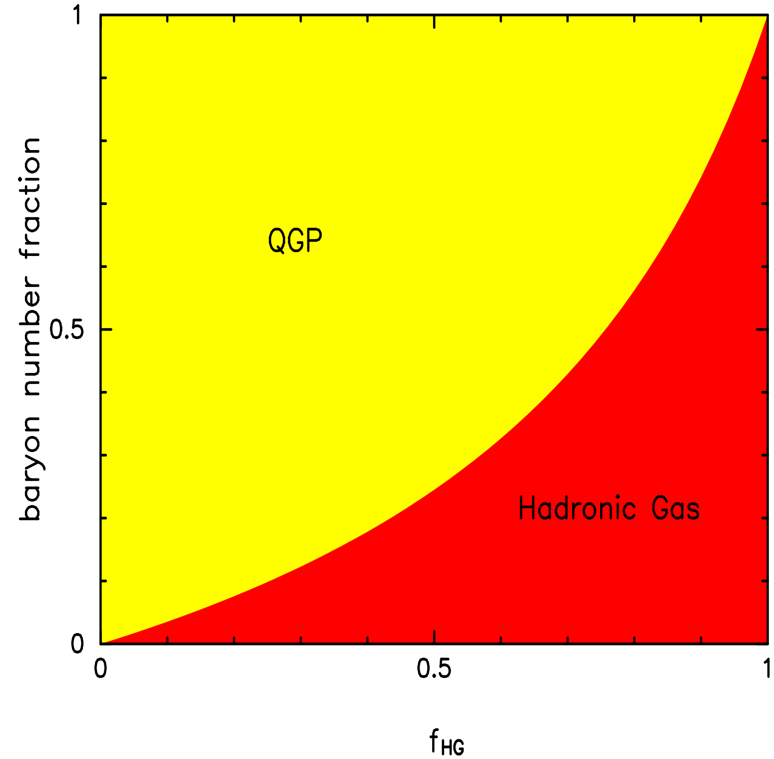
$$Q = 0 = n_Q^{\rm QGP} V_{\rm QGP} + n_Q^{\rm HG} V_{\rm HG} = V_{\rm tot} \left[(1 - f_{\rm HG}) n_Q^{\rm QGP} + f_{\rm HG} n_Q^{\rm HG} \right[$$

where the total volume $V_{\rm tot}$ is irrelevant to the solution. Analogous expressions can be derived for $L - B$ and S/B constraints.

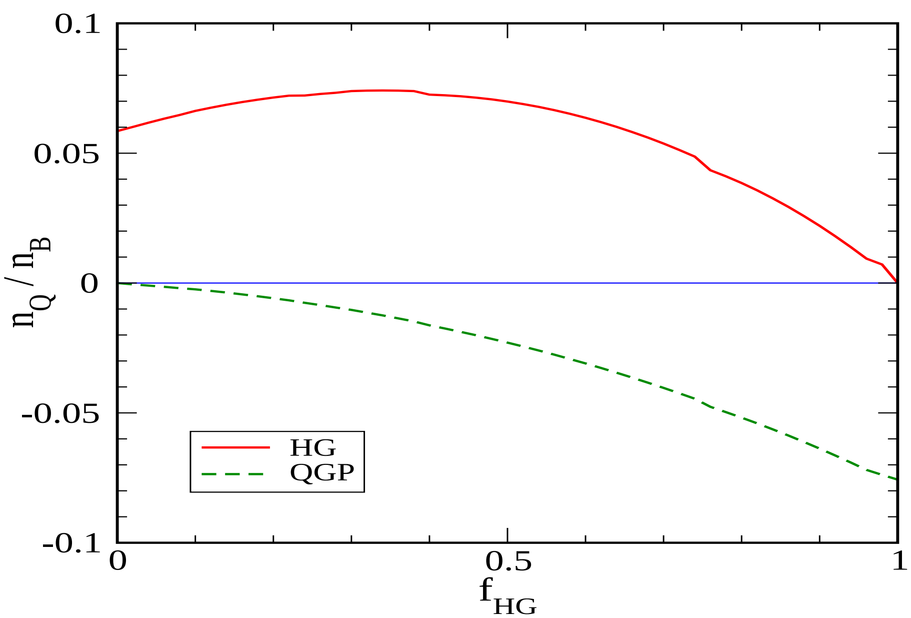
In following we assume that mixed phase exists $10\,\mu s$ and that $f_{\rm HG}$ changes linearly in time. Actual values will require dynamic nucleation and transport theory description.

Charge and baryon number distillation

Initially at $f_{\mathrm{HG}} = 0$ all matter in QGP phase, as hadronization progresses with $f_{\mathrm{HG}} \to 1$ the baryon component in hadronic gas reaches 100%.



The constraint to a charge neutral universe conserves SUM of charges in both fractions. Charge in each fraction can be and is non-zero.

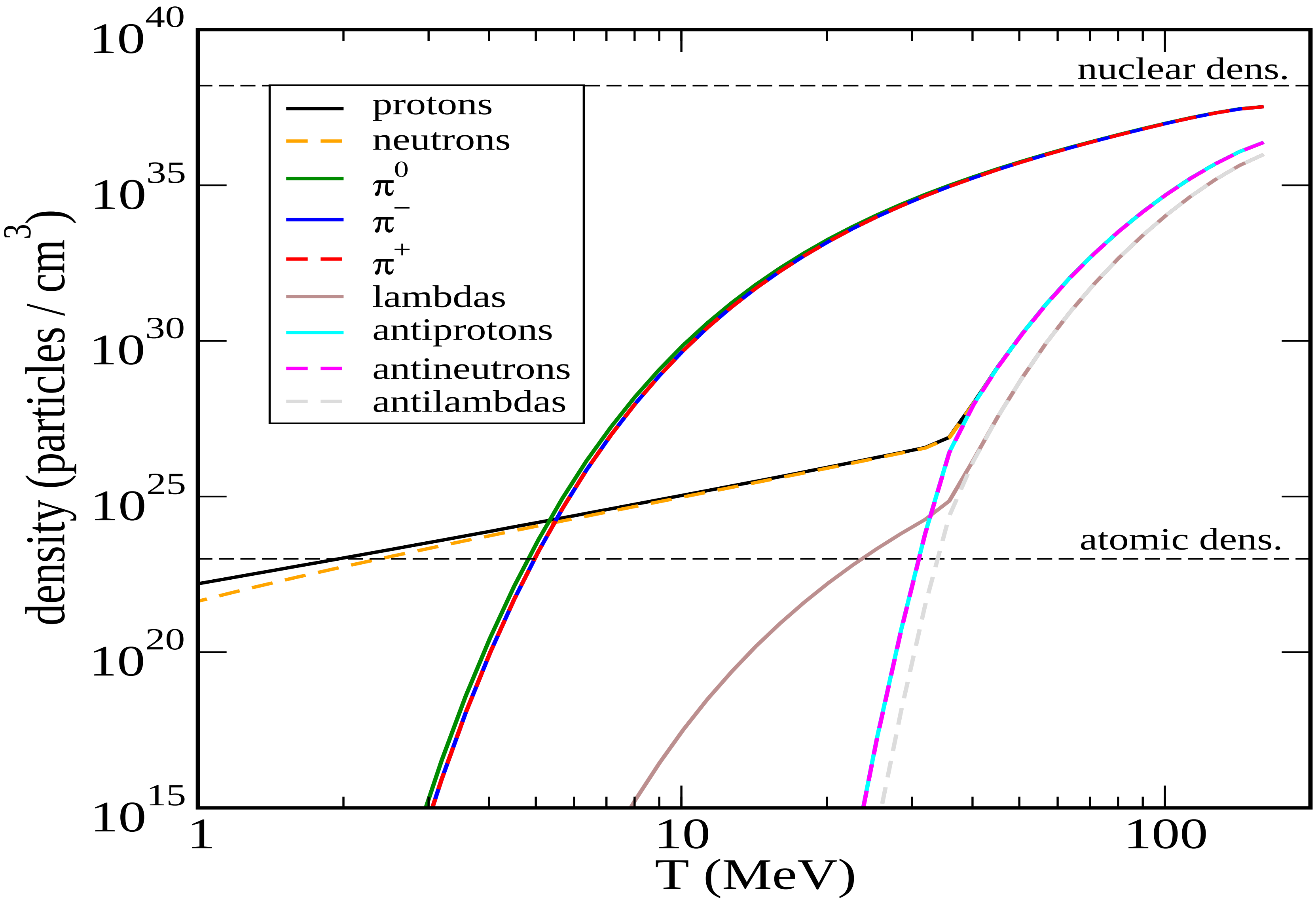


Even a small charge separation between phases introduces a finite non-zero local Coulomb potential and this amplifies any existent baryon asymmetry (protons vs antiprotons).

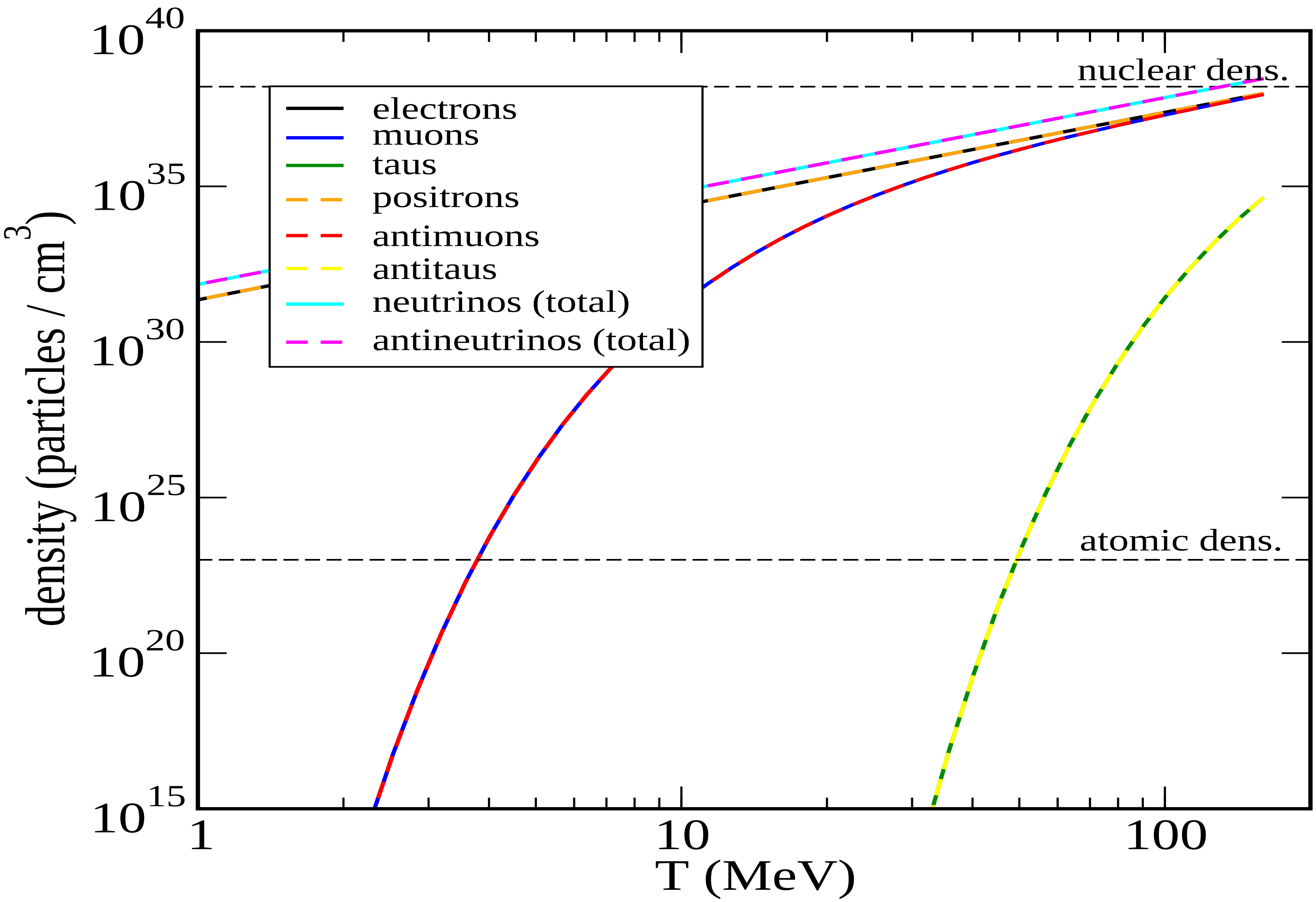
This mechanism noticed by Witten in his 1984 paper, and exploited by A. Olinto for generation of magnetic fields.

AMPLIFICATION OF A SMALL B–$\overline{B}$ ASYMMETRY.

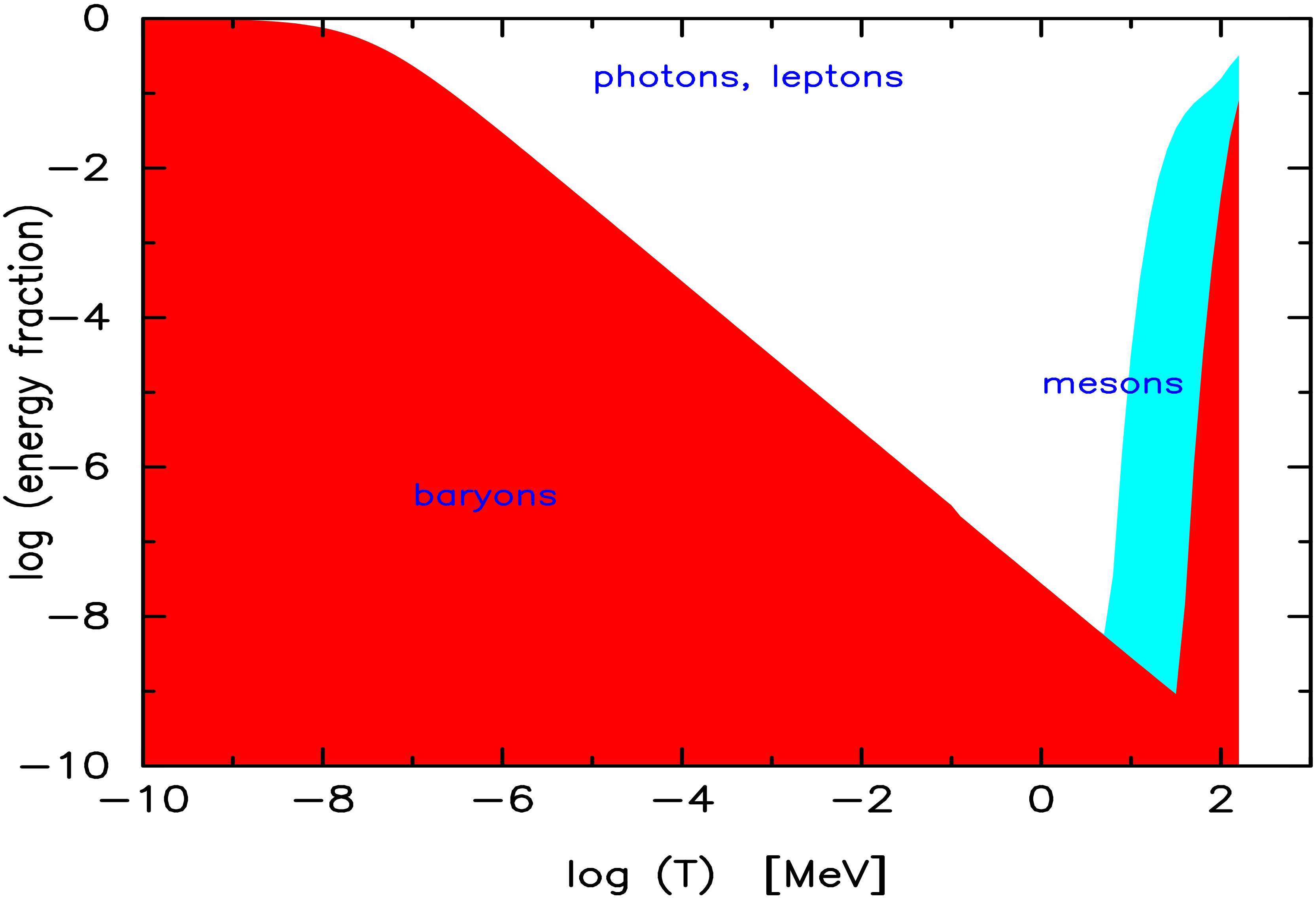
Hadronic Particle Densities



Lepton Densities

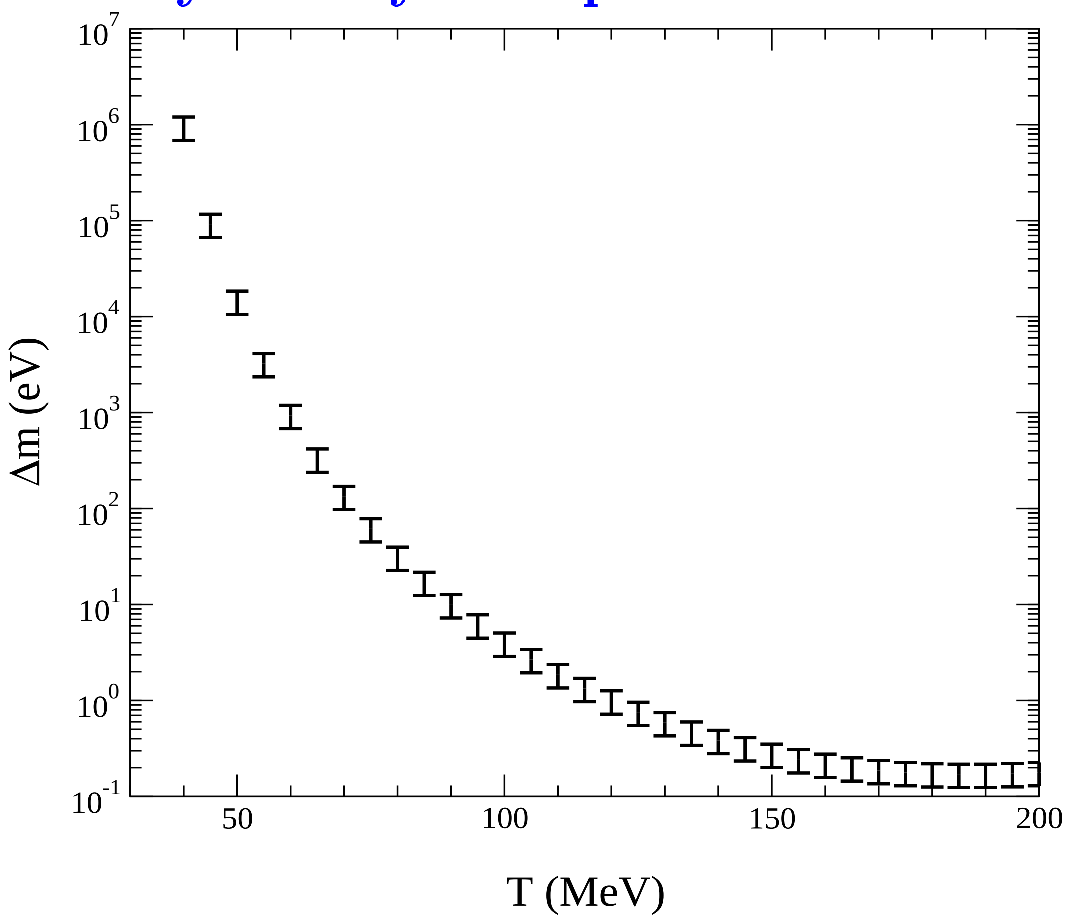


Energy in luminous hadronic Universe



Other source of baryon asymmetry

If CPT symmetry is broken, the finite baryon number could be supported by resulting particle-antiparticle mass asymmetry. In PDG the hadron limit on CPT mass differences is at the level of many eV: $|M_P - M_{\bar{P}}| < 55$ eV, (but $|M_e - M_{\bar{e}}| < 4\,10^{-3eV}$). This is MUCH larger than the baryochemical potential required to sustain the asymmetry near phase transformation.



Nucleon-antinucleon mass difference required to describe the baryon number in the Universe at zero chemical potential

We found a better CPT limit on $m - \overline{m}$

Consider K_L and K_S states, in the standard formalism:

$$K_L = \frac{1}{\sqrt{2+2\epsilon^2}}\left[(1+\epsilon)K^0 + (1-\epsilon)\bar{K^0}\right]\ ,\ K_S = \frac{1}{\sqrt{2+2\epsilon^2}}\left[(1-\epsilon)K^0 - (1+\epsilon)\bar{K^0}\right]\ ,$$

where $K^0 = |d\bar{s}\rangle$, $\bar{K^0} = |\bar{d}s\rangle$, $\epsilon = 2.1 \pm 0.3 \times 10^{-3}$ violates CP-symmetry.

We express the (assumed) CPT-violating mass difference between quarks and antiquarks as:

$$m_{s,\bar{s}} = m_s^0 \pm \frac{\delta m_s}{2} \qquad m_{d,\bar{d}} = m_d^0 \pm \frac{\delta m_d}{2}\ ,$$

where the signs of δm_s and δm_d are undetermined. The mass difference between K_L and K_S becomes:

$$m_{K_L} - m_{K_S} \equiv \Delta m = \Delta m_w + 2\,\epsilon f\,\left[\,(m_{\bar{s}} - m_s) - (m_{\bar{d}} - m_d)\,\right] = 3.49 \pm 0.01 \times 10^{-6}\,\textbf{eV}$$

Δm_w is the second order WI mass difference,
it agrees well with measured value.
$f \simeq 1$ expresses the response of Kaon mass to a small change in quark masses.

This means:

$$|(m_{\bar{s}} - m_s) - (m_{\bar{d}} - m_d)| \ll \frac{\Delta m}{2\epsilon f} \approx 10^{-3}\,\mathrm{eV}\ .$$

We find that the current upper limit to the mass difference between quarks and antiquarks in the d and s flavors is $\ll 10^{-3}\,\mathrm{eV}$ if the magnitude of the CPT violation is uncorrelated across flavors. In this case, the relative precision with which the strange quark mass difference is determined appears to be by far the most precise such value presently known:

$$\left|\frac{m_s - m_{\bar{s}}}{m_s + m_{\bar{s}}}\right| \ll 10^{-11}\,,$$

providing a strong constraint for any CPT model considered. Also, we excluded the possibility that the quark/antiquark mass (hadron/ antihadron mass) difference is associated with baryon asymmetry in the early Universe.

FINAL REMARKS

Theoretical study of the early Universe beginning at $t = 10\,\mu s$

Control of chemical potentials, particle abundances

Distillation of baryon number in domains of the Universe

Strangeness in early Universe under study (Strangeletts)

CPT violation too small to influence the understanding of early Universe

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We just begin to transfer the 'know-how' from the study of nuclear collisions to the study of the early Universe

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