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LONG-RANGE VACUUM CORRELATIONS?

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We obtain the polarization of the heavy quark condensate induced by Maxwell fields. We then describe a possible line of approach to the problem of light quark condensate polarization and long-range order induced by an external Maxwell field.

1 Overview

Quarks, being dual carriers of both 'color' and 'electric' charges, serve as a bridge between strong and electromagnetic interactions. The decay anomaly of $\pi^0 \rightarrow \gamma+\gamma$ is the well known example of an induced photon-hadron coupling. We address in our ongoing work the more complex circumstance of induced photon-gluon couplings [1] and more specifically the resulting consequences for the structure and properties of the vacuum. The object of our interest is the macroscopic electromagnetic field and its capability to induce long-range order among quark fluctuations [2]. In a loose sense Maxwell field influence on virtual matter can be compared to the effect a Maxwell potential has applied to a material body: depending on the character of the substance (conductor, semiconductor, insulator) very different families of long range effects arise. We recall that from the perspective of QCD-color interactions the vacuum is an excellent insulator, no real color charge is present. From perspective of QED-electric charge we have a semi-conductor; when we overcome the potential gap $V = 2m$, free charges arise as was first noted in Schwinger's seminal work [3].

A practical model of the QCD vacuum of strong interactions involves random gluon fluctuations [4], which can be seen to be the origin of the quark fluctuations. For the case of heavy quarks an in-principle solution is in fact available describing the deformation of the vacuum by Maxwell fields, and we present the formal result. However, its consequences have so far not been explored. Even when evaluated in more detail, this result does not answer the question we pose about long range correlations, but it allows to gather some insight about the interplay of the QED-QCD phenomena due to quarks.

For light quark fluctuations we presented at the meeting a possible scheme of approach which is here briefly described in words: the combination of random classical gluon condensate fields in presence of externally applied long-range Maxwell fields has the mathematical structure of a random walk problem in proper time.

2 Heavy quark condensate

The phenomenon of Fermi field condensation is already developed by Schwinger, who in his seminal paper on gauge invariance and vacuum fluctuations notes [see [3], Eqs. (2.10), (5.2)]:

$$\langle\bar{\psi}(x)\psi(x)\rangle \equiv \langle\frac{1}{2}[\bar{\psi}(x),\psi(x)]\rangle \equiv \lim_{t'\to t}\langle T(\psi(x)\bar{\psi}(x'))\rangle \equiv \Sigma = -\frac{\partial\Gamma}{\partial m}\,. \qquad (1)$$

The left-hand side defines precisely the meaning of the quark condensate in terms of the Fermi field operators at equal space-time points. The right-hand-side of Eq. (1) refers to the effective action density $\Gamma[a_\mu, b_\mu]$ of fermions in presence of Maxwell U(1) a_μ and chromo-SU(3) gauge potentials b_μ. Adopting the stochastic vacuum picture of QCD [4], both these fields are classical.

In principle, the chromo-potentials b_μ are dominant and random, but a solution of such a problem is not in sight yet (see next section) for such a configuration. At first we pursue the case of arbitrarily strong, but on fermion Compton wavelength scale constant fields. Schwinger obtained up to first order in Fermi loop expansion (denoted here by superscript (1)):

$$-m\frac{1}{2}\langle[\bar{\psi}(x),\psi(x)]\rangle^{(1)} = \frac{m^2}{4\pi^2}\int_0^\infty \frac{ds}{s^2}\mathrm{e}^{-m^2 s}\left[\frac{sE}{\tan sE}\,\frac{sB}{\tanh sB} - 1\right]\,. \qquad (2)$$

In Eq. (2) the subtraction removes the contribution to the condensate from the noninteracting vacuum, corresponding to the effect of normal-ordering of operators with respect to the vacuum state in which the computation of the matrix elements is completed. We note that the right-hand side is even in m, E, B.

The electric-magnetic-field-like invariants E, B are related to the gauge potentials by:

$$B^2 = \frac{1}{2}\sqrt{(\vec{E}^{\,2}-\vec{B}^{\,2})^2+4(\vec{E}\cdot\vec{B})^2} - \frac{1}{2}(\vec{E}^{\,2}-\vec{B}^{\,2}) \to |\vec{B}|^2\,,\ \text{for } |\vec{E}|\to 0\,; \quad (3)$$

$$E^2 = \frac{1}{2}\sqrt{(\vec{E}^{\,2}-\vec{B}^{\,2})^2+4(\vec{E}\cdot\vec{B})^2} + \frac{1}{2}(\vec{E}^{\,2}-\vec{B}^{\,2}) \to |\vec{E}|^2\,,\ \text{for } |\vec{B}|\to 0\,. \quad (4)$$

where:

$$\vec{E}^{\,2}-\vec{B}^{\,2} \equiv -\frac{1}{2}\mathrm{Tr}_c F_{\mu\nu}F^{\mu\nu} \equiv -\frac{1}{2}F^2\,,\ \vec{E}\cdot\vec{B} = -\frac{1}{4}\mathrm{Tr}_c F_{\mu\nu}\tilde{F}^{\mu\nu} \equiv -\frac{1}{4}F\tilde{F}\,. \quad (5)$$

Both E, B fields combine Maxwell a_μ fields of charge e and chromo-fields b_μ with charge g:

$$F^{\mu\nu} = \partial^\mu e a^\nu - \partial^\nu e a^\mu + \partial^\mu g b^\nu - \partial^\nu g b^\mu + f^{ijk} g^2 b_j^\mu b_k^\nu \,, \qquad \tilde{F}^{\mu\nu} \equiv \frac{1}{2}\epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta} \,. \tag{6}$$

The color indices are not made explicit at all times, and their sums Tr_c are also mostly left implicit.

We will next consider Eq. (2) keeping in mind that at best it applies to the condensation of heavy quarks, such as charm, for which the vacuum gluon fluctuations are effectively a constant field. In absence of Maxwell fields the sign of the gluon field condensate is suggesting a configuration with $E \to 0$ for the vacuum fluctuations relative to normal ordered operators. Namely, Lorentz and gauge symmetry of the vacuum state dictate that the VEV of a product of two field operators satisfies:

$$\langle F^a_{\mu\nu}(x) F^b_{\rho\sigma}(x)\rangle = (g_{\mu\rho} g_{\nu\sigma} - g_{\mu\sigma} g_{\nu\rho})\delta^{ab} \frac{1}{96} \langle F^2(x)\rangle \,, \tag{7}$$

Taking the required contractions and using the first of Eqs. (5), one finds:

$$\langle \sum_a \vec{B}_a^{\,2} \rangle = -\langle \sum_a \vec{E}_a^{\,2} \rangle \tag{8}$$

Since:

$$\langle \frac{1}{4\pi^2} F^2 \rangle \simeq (2.3 \pm 0.3) 10^{-2} \mathrm{GeV}^4 = [390 \pm 12\,\mathrm{MeV}]^4 \,, \tag{9}$$

we know that $\langle \vec{B}_a^{\,2}(x)\rangle$ is positive, and $\langle \vec{E}_a^{\,2}(x)\rangle$ is negative. However, these signs are defined with reference to the perturbative state $|P\rangle$. Specifically, we are considering products of field operators which are normal-ordered with respect to $|P\rangle$ – where the condensate fields thus vanish by definition. The interpretation of Eq. (8) is that the B-field fluctuates in the true QCD vacuum $|V\rangle$ with a bigger amplitude than in the perturbative 'vacuum' $|P\rangle$, while the E-field fluctuates with a smaller amplitude than in $|P\rangle$: $|V\rangle$, the true vacuum, may be completely 'magnetic', without any electric fluctuations, while $|P\rangle$ may be completely 'electric', without any magnetic fluctuations.

In the 'magnetic' limit the glue fluctuations induce a heavy quark condensate according to:

$$-m\frac{1}{2}\langle[\bar{\psi}(x), \psi(x)]\rangle^{(1)} = \frac{m^2}{4\pi^2} \int_0^\infty \frac{ds}{s^2} \mathrm{e}^{-m^2 s} \left[\frac{sB}{\tanh sB} - 1\right] \,. \tag{10}$$

This expression can be studied in the perturbative limit, assuming that the mass of the quark is arbitrarily large. To this end one expands in powers of B:

$$\frac{x}{\tanh x} - 1 = \frac{x^2}{3} - \frac{x^4}{45} + \frac{2x^6}{945} \ldots . \tag{11}$$

Remarkably, the first term in the series leads to an m-independent contribution:

$$-m\frac{1}{2}\langle[\bar\psi(x), \psi(x)]\rangle^{(1)} \to \frac{B^2}{12\pi^2} - \frac{1}{90\pi^2}\frac{B^4}{m^4} + \ldots . \tag{12}$$

The small parameter of this expansion is B^2/m^4. We identify the vacuum field as the gluon condensate:

$$B^2 \to \frac{1}{2}\langle V|F^2|V\rangle . \tag{13}$$

Eq. (12) stands at the beginning of the sum rule determination of glue condensates using heavy quark bound states and production cross sections [5].

3 Constant Maxwell fields and heavy quark condensate

When we consider along with QCD vacuum fluctuations the Maxwell field, we cannot find a purely magnetic vacuum configuration, thus we have both $B \neq 0$ and $E \neq 0$. We separate in Eq. (2) the fluctuations due purely to E and B fields and write:

$$-m\frac{1}{2}\langle[\bar\psi(x), \psi(x)]\rangle^{(1)} = \frac{m^2}{4\pi^2}\int_0^\infty \frac{ds}{s^2} e^{-m^2 s}\left\{\left[\frac{sE}{\tan sE} - 1\right]\cdot\left[\frac{sB}{\tanh sB} - 1\right] + \left[\frac{sB}{\tanh sB} - 1\right] + \left[\frac{sE}{\tanh sE} - 1\right]\right\} . \tag{14}$$

The term with B-field only is the condensate prior to application of the perturbing field, and the term with E-field only is the analogue of the Maxwell-electric field expression. The first term in Eq. (14) is the interference between Maxwell and chromo fields we consider. We use the representations:

$$\frac{x}{\tan x} = 1 + 2x^2\sum_{k=1}^{\infty}\frac{1}{x^2 - (k\pi)^2} , \qquad \frac{x}{\tanh x} = 1 + 2x^2\sum_{k=1}^{\infty}\frac{1}{x^2 + (k\pi)^2} . \tag{15}$$

Identifying $s = xk\pi/E, s = xk\pi/B$, as appropriate, we obtain as the effect on the condensate due to the presence of both E, B:

$$\begin{aligned} &- m\frac{1}{2}\langle[\bar\psi(x), \psi(x)]\rangle^{(1)} + m\frac{1}{2}\langle[\bar\psi(x), \psi(x)]\rangle^{(1)}|_{E=0} + m\frac{1}{2}\langle[\bar\psi(x), \psi(x)]\rangle^{(1)}|_{B=0} \\ &= \frac{m^2}{\pi^2}\sum_{k=1}^{\infty}\sum_{n=1}^{\infty}\int_0^\infty ds \frac{s}{s^2 + (n\pi/B)^2}\; e^{-sm^2}\frac{s}{s^2 - (k\pi/E)^2 - i\varepsilon} . \end{aligned} \tag{16}$$

In principle, E-fields allow pair production and hence the pole in the corresponding term is found on the real axis. Since the representations given above are absolutely convergent, arbitrary reorganization of the terms can be attempted, in order to evaluate the integral. Unfortunately, despite its simplicity, even this simple example is analytically nontrivial. Moreover, we really would like to explore the limit of small quark masses in which case the short-range character and fluctuating nature of the glue condensate has to be considered.

4 Fokker-Planck dynamics in the vacuum

While our efforts continue to understand the non-perturbative character of the QCD vacuum condensate subject to external fields, it became clear that the interesting question is not so much how the condensate behaves in presence of strong Maxwell fields, but if it acquires (possibly small) long-range ordering. To put it differently, if the range of the quark condensate is about [6] 0.3 fm, we would like to isolate the magnitude of the component that has longer range possibly arising due to the presence of a long-range Maxwell field. We argue here that this problem could be solved considering diffusion in a 1+4 dimensional geometry, where the added dimension in which the dynamics occurs is the 'proper time' used in Schwinger's paper [3].

Considering that the work is not complete, let us outline in short just the proposed idea: Schwinger has reduced the problem of obtaining effective action in one loop approximation to finding a solution of Heisenberg's equations of motion of a quasi-quantum mechanical problem formulated in 1+4 dimensions. In the past 50 years no other soluble example has been found beyond the case of a variety of constant fields. However, re-inspecting these equations to understand better the dynamics, we realized that the nature of the solutions of Schwinger is that (dependent on sign of the field) particles experience either drag or accelerating forces. Since we are dealing with a tensor force, and quantum rather than classical dynamics, there are many mathematical complications to consider, but in the end we are simply considering particle motion in a background force field of trivial nature.

Now, enter the QCD-random vacuum chromo field, and in first instance let us turn off the Maxwell field. The problem changes entirely in its physical nature: the test particle is now diffusing in the random fields. Its path is highly irregular, it makes no sense to solve Heisenbergs equation of motion, even if we could. Instead of looking for the position and momentum of the quasi-dynamics in 5 dimensions, we rather should seek to evaluate the probability distribution in the large (4+4)-dimensional phase space as function of proper time.

Such random dynamics is described by a Fokker-Planck equation and it is evident inspecting the dynamical equations in proper time that the fluctuations in the vacuum lead to a diffusion constant which is the glue condensate. In such a visualization one clearly recognizes that the heavy quark dynamics is simpler, while light quarks respond in a more volatile way to vacuum forces.

Introduction of a constant background field introduces a drag/acceleration on the virtual quarks, which competes with the random walk, and extends the range of correlations. Since the ordered field distinguishes quarks and anti-quarks, the proper space distributions of these particles will differ.

To recapitulate, we have presented analytic solution to the problem of polarization of heavy quark condensate by Maxwell fields. We have argued that an externally applied Maxwell field acts to order the random fluctuations of the quark condensate, potentially inducing a long-range order in the vacuum.

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