Special Relativity and Strong Fields: A personal perspective

Presented by Johann Rafelski

THE UNIVERSITY OF ARIZONA

Institute of Theoretical Physics

Uniwersystet Wroclawski

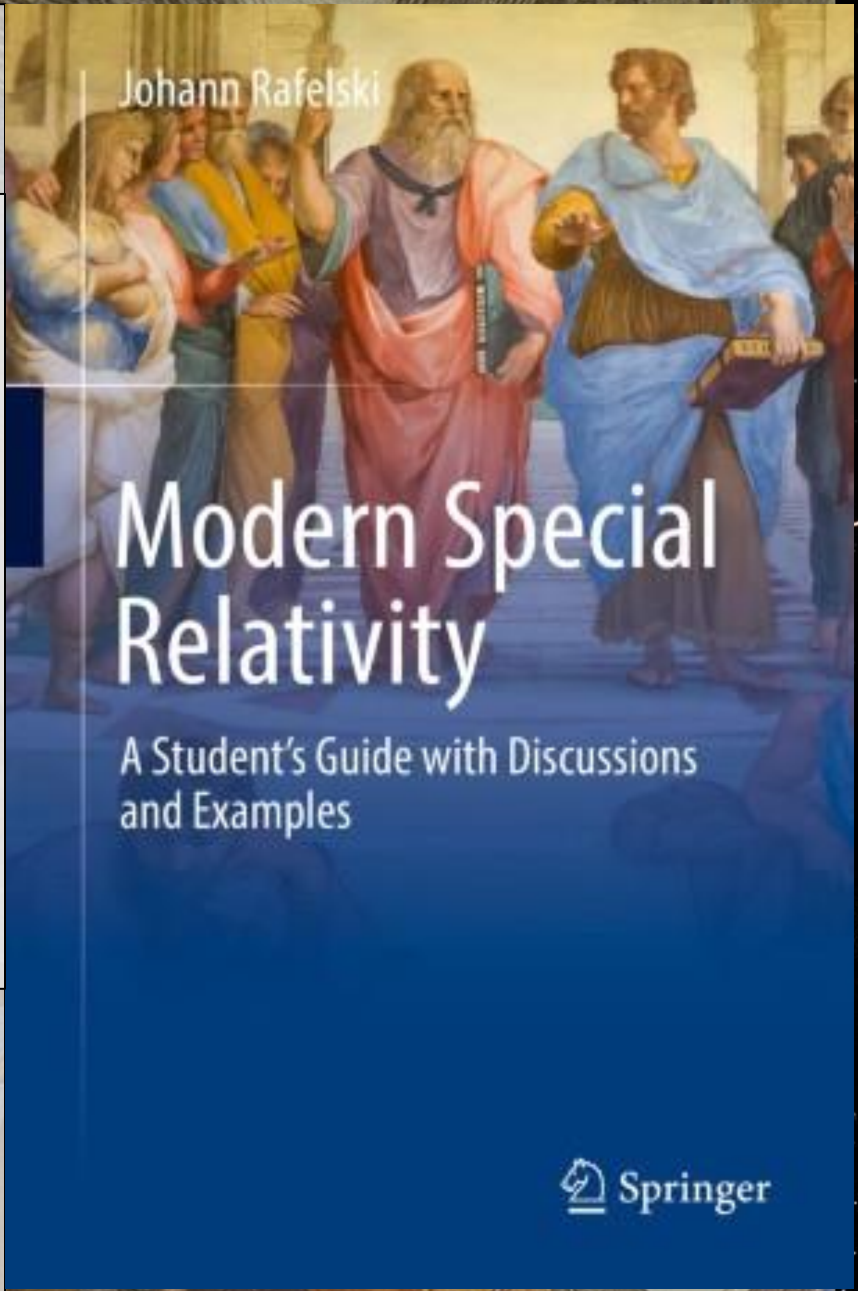
April 1st, 2022

Setting the stage:

1. **How do we teach SR well?**

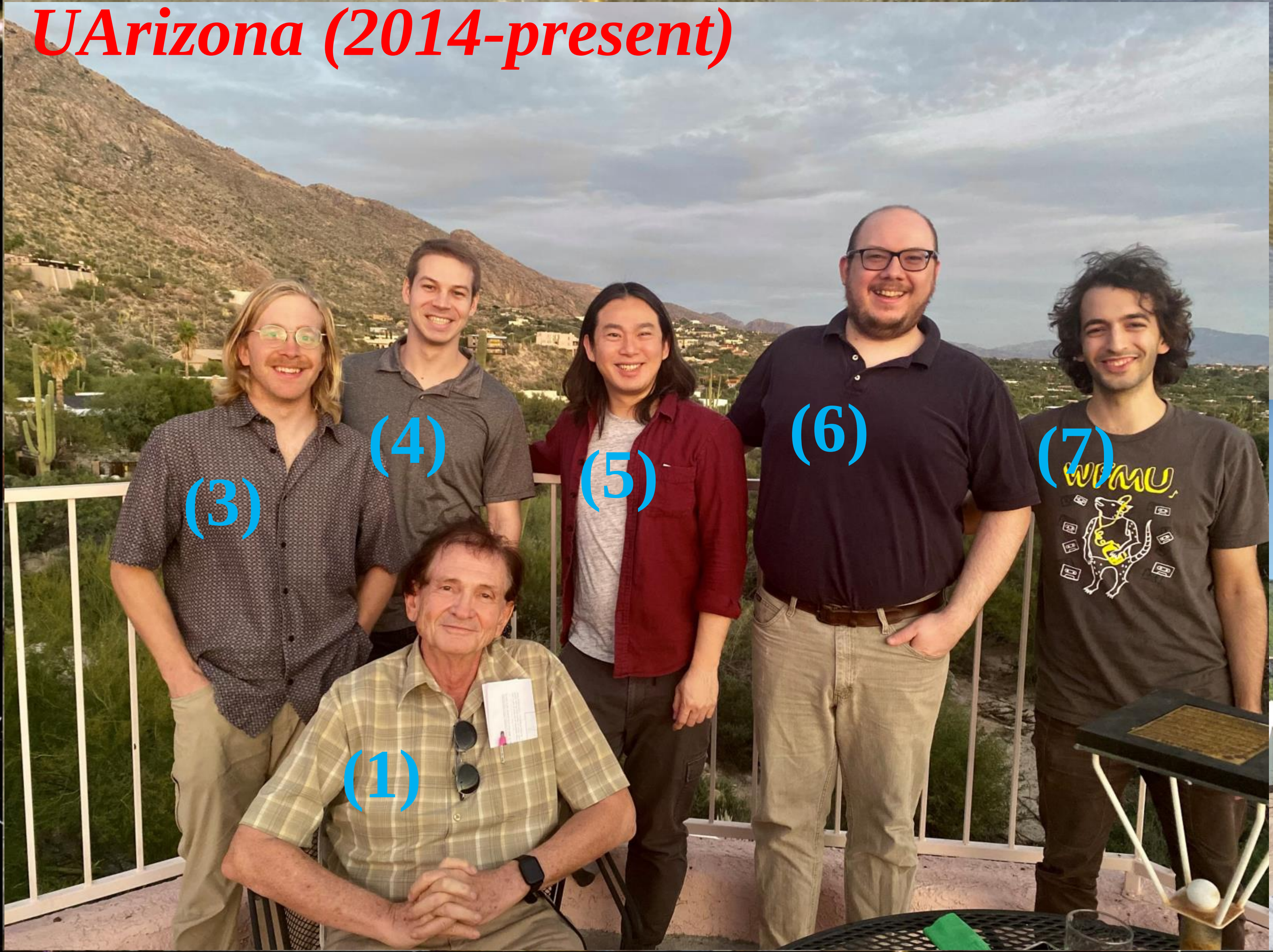
The following frontier domains of physics are demanding researchers very well trained in SR. However, SR is poorly represented in introductory textbook used for most part by "out of research field lecturers". The unfinished understanding of SR when acceleration is present contributes to student confusion. The book **"Modern Special Relativity"** provides historical background as a motivation to return to the topic and aims at a very elementary level to resolve some of the misunderstandings while motivating students to embark on study of the acceleration frontier.

2. **How to handle (strong) acceleration in SR?**
3. **How do we complete E&M?**
4. **Is there a more fundamental meaning of acceleration?**
5. **Highlights of our recent research results.**



Research Group at Tucson, Arizona

1. **Dr. Johann Rafelski**
2. **Dr. Martin Formanek**
3. **Chris Grayson**
4. **Will Price**
5. **Cheng Tao Yang**
6. **Andrew Steinmetz**
7. **Stefan Evans**



My lifelong interest in Special Relativity and Strong Fields

Strong forces imply strong acceleration creating new challenges

Einstein developed SR invoking only inertial observers. The word acceleration does not appear in his 1905 work. Is the Lorentz force complete?

In daily life, all accelerations are far below the natural "unit-1" value of acceleration.

$$a_{cr} = m_e c^2 \frac{c}{\hbar} = 2.33 \times 10^{29} \frac{\mathrm{m}}{\mathrm{s}^2}$$

This is also the acceleration generated by Schwinger "critical" EM fields:

$$E_{cr} = \frac{(m_e c^2)^2}{e\hbar c} = 1.323 \times 10^{18} \frac{\mathrm{V}}{\mathrm{m}}$$

$$B_{cr} = \frac{(m_e c^2)^2}{e\hbar c^2} = 4.414 \times 10^{9}\ \mathrm{T}$$

SR absorbs nano-acceleration setting $\Delta v = a\Delta t$ but Langevin was clear: Accelerated twins age slower compared to inertial twins.

Ultra-relativistic electron in a magnetic field of 4.41 T at CERN:

$$a_{CERN} = \left(\frac{e}{m_e}\right) \boldsymbol{v} \times \boldsymbol{B} = 2.33 \times 10^{20} \frac{\mathrm{m}}{\mathrm{s}^2} \sim \textbf{nano } \boldsymbol{a_{cr}}$$

Classical Electromagnetism is incomplete!

We have two separate theories:

- Given sources of charges and currents, calculate EM fields.
- Given EM fields, calculate charged particle motion.

"… a complete satisfactory treatment of the reactive effects of radiation does not exist."
– J. D. Jackson, *Classical Electrodynamics*, p. 781, (1999).

There is a disconnect as accelerated charges radiate and lose energy and momentum which should be reflected in their motion! A self-consistent reaction/friction force is needed.

There are many models of radiation friction, but no action principle.
To solve the problem, we need to connect acceleration and SR.

Maybe acceleration is not what we think: Connecting temperature and acceleration

Strong Fields → ***Temperature*** ← ***Acceleration***

Interpretation of external fields as temperature

Temperature representation of Euler-Heisenberg action in electric-dominated fields.

Notes on black-hole evaporation

Thermal background (Unruh temperature) experienced by an observer undergoing constant acceleration in a field-free vacuum.

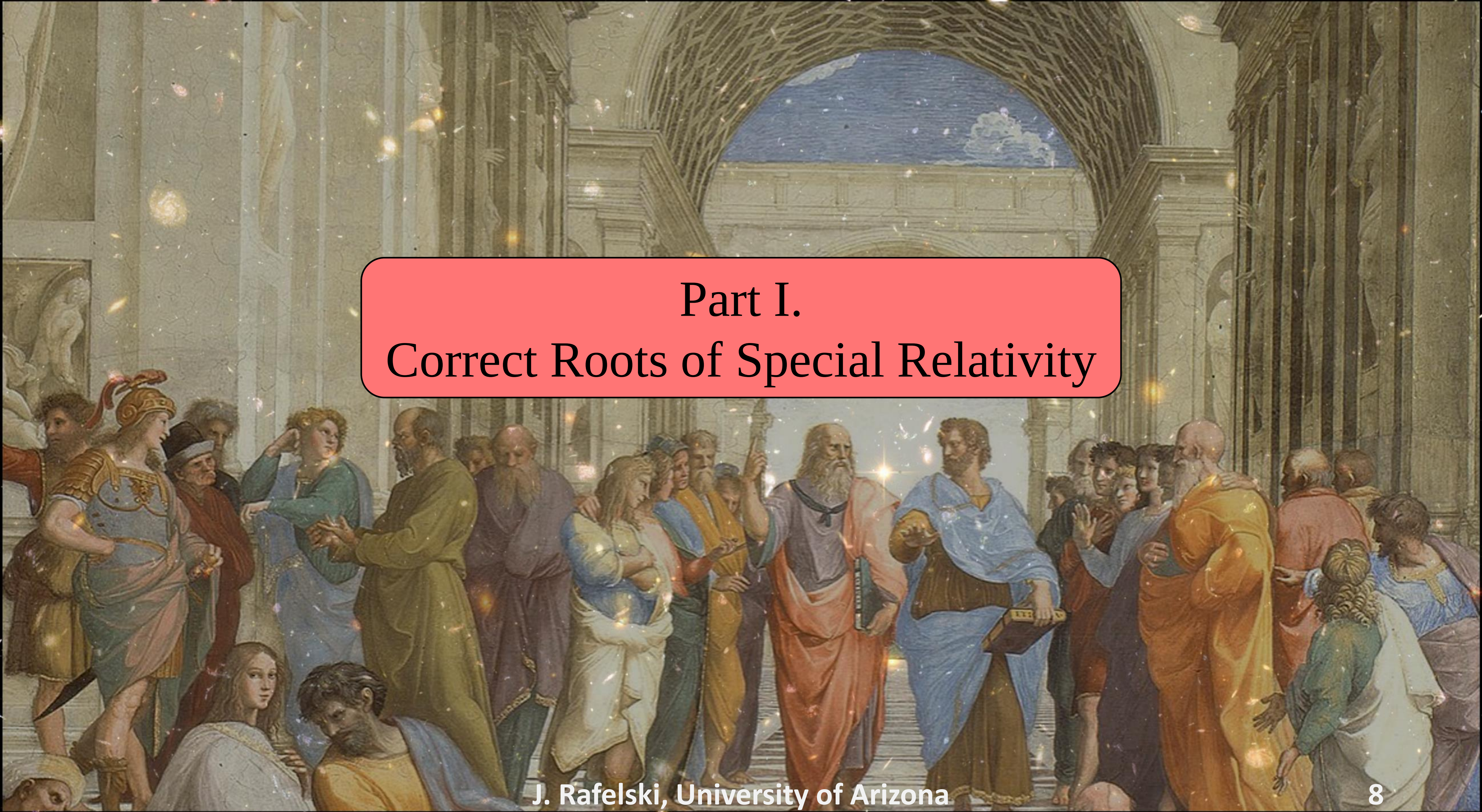
W. H. Unruh

Gravity Swing, Taipei 101, (2012)

B. Müller, W. Greiner, and J. Rafelski. "Interpretation of external fields as temperature.' Physics Letters A 63.3 (1977)

W. G. Unruh, "Notes on black-hole evaporation." Physical Review D 14.4 (1976)

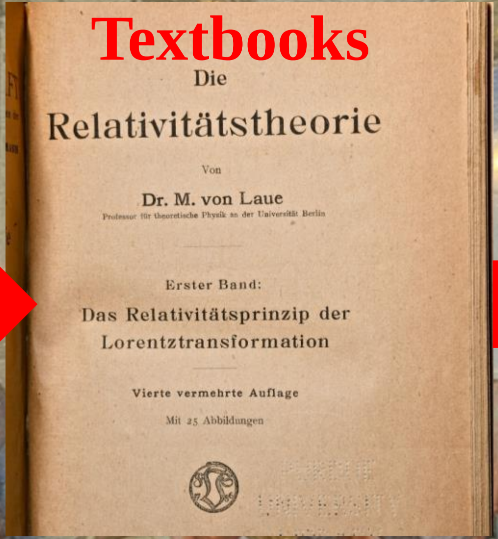
L. Labun and J. Rafelski, "Acceleration and vacuum temperature." Phys. Rev. D 86, 041701(R) (2012)



Part I.
Correct Roots of Special Relativity

To understand principles of special relativity, it is best to look at original work. Why?

Telephone game with Lorentz-FitzGerald body contraction:



Die

Relativitätstheorie

Von

Dr. M. von Laue

Professor für theoretische Physik an der Universität Berlin

Erster Band:

Das Relativitätsprinzip der Lorentztransformation

Vierte vermehrte Auflage

Mit 25 Abbildungen

"Body is contracted."
Correct.

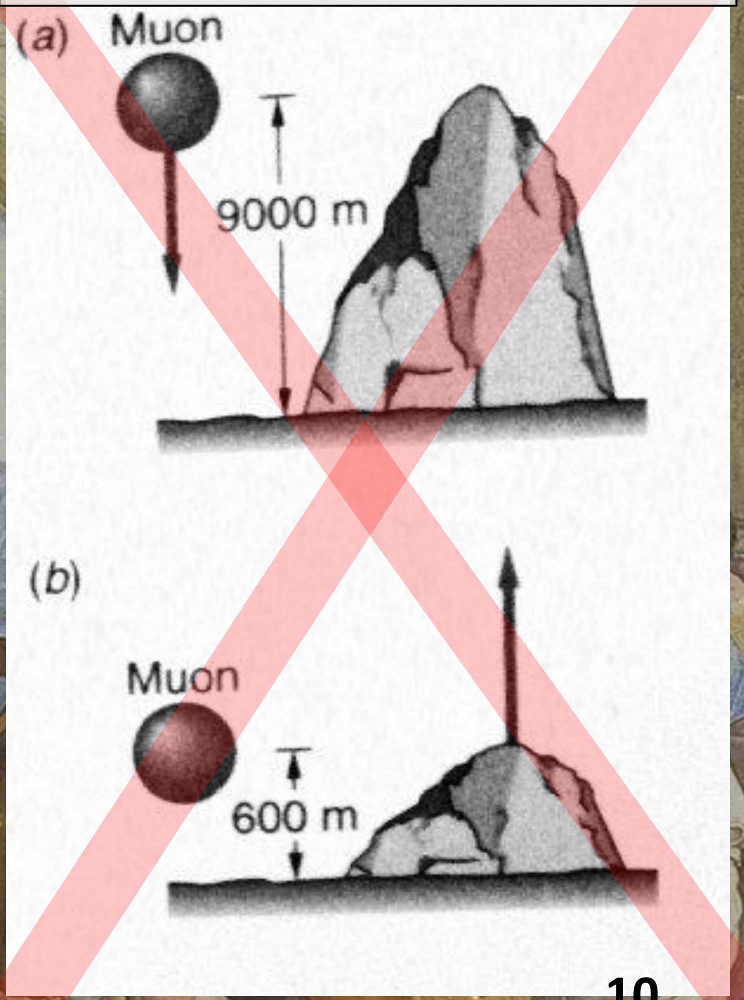
"Length is contracted."
Correct, but requires context.

"Distance is contracted."
Misleading.
↓
"Space is contracted."
Junk!

SR teacher facing students

- Books claims Lorentz contraction and time dilation are the same and that one confirms the other. This is wrong.
- Books claim that SR has "paradoxes" or "not real" effects whereas frame-dependant phenomena are well established in other areas of physics.
- Students fact-check you live against **internet prophets** and their **wrong but entertaining videos.**
- I think SR is a living and evolving theory while SR is taught as a footnote of GR.

From a serious book that millions of students have used.



390 SCIENCE. [VOL. XIII. No. 328

LETTERS TO THE EDITOR.

***Correspondents are requested to be as brief as possible. The writer's name is in all cases required as proof of good faith.*

The editor will be glad to publish any queries consonant with the character of the journal.

Twenty copies of the number containing his communication will be furnished free to any correspondent on request.

The Ether and the Earth's Atmosphere.

I HAVE read with much interest Messrs. Michelson and Morley's wonderfully delicate experiment attempting to decide the important question as to how far the ether is carried along by the earth. Their result seems opposed to other experiments showing that the ether in the air can be carried along only to an inappreciable extent. I would suggest that almost the only hypothesis that can reconcile this opposition is that the length of material bodies changes, according as they are moving through the ether or across it, by an amount depending on the square of the ratio of their velocity to that of light. We know that electric forces are affected by the motion of the electrified bodies relative to the ether, and it seems a not improbable supposition that the molecular forces are affected by the motion, and that the size of a body alters consequently. It would be very important if secular experiments on electrical attractions between permanently electrified bodies, such as in a very delicate quadrant electrometer, were instituted in some of the equatorial parts of the earth to observe whether there is any diurnal and annual variation of attraction, — diurnal due to the rotation of the earth being added and subtracted from its orbital velocity; and annual similarly for its orbital velocity and the motion of the solar system.

GEO. FRAS. FITZ GERALD.

Dublin, May 2.

Lorentz-FitzGerald Body Contraction, 1889

Body contraction in the direction of motion was first described by FitzGerald in 1889.
Lorentz once made aware, called it "FitzGerald body contraction." FitzGerald who passed away before SR was fully developed could not defend his priority.

Restatement of FitzGerald text:
"We know that electric forces are affected by the motion of the electrified bodies relative to the ether, and it seems not an improbable supposition that the molecular forces are affected by the motion, and that the size of a body alters consequently."

"On the Electrodynamics of Moving Bodies" A. Einstein, 1905

"Does the Inertia of a Body Depend upon its Energy Content?" A. Einstein, 1905

3. ***Zur Elektrodynamik bewegter Körper;***
von A. Einstein.

Daß die Elektrodynamik Maxwells — wie dieselbe gegenwärtig aufgefaßt zu werden pflegt — in ihrer Anwendung auf bewegte Körper zu Asymmetrien führt, welche den Phänomenen nicht anzuhaften scheinen, ist bekannt. Man denke z. B. an die elektrodynamische Wechselwirkung zwischen einem Mag-

13. ***Ist die Trägheit eines Körpers von seinem Energieinhalt abhängig?***
von A. Einstein.

Die Resultate einer jüngst in diesen Annalen von mir publizierten elektrodynamischen Untersuchung[1]) führen zu einer sehr interessanten Folgerung, die hier abgeleitet werden soll.

As Einstein titles/content imply SR is about:

- Electromagnetism.
- Body contraction.
- Time dilation.
- $E = mc^2$
- Constancy of the speed of light.

Einstein in 1905 SR introduced the principle of relativity into EM and explored consistency consequences.

SR is not about gravity and/or space-time.

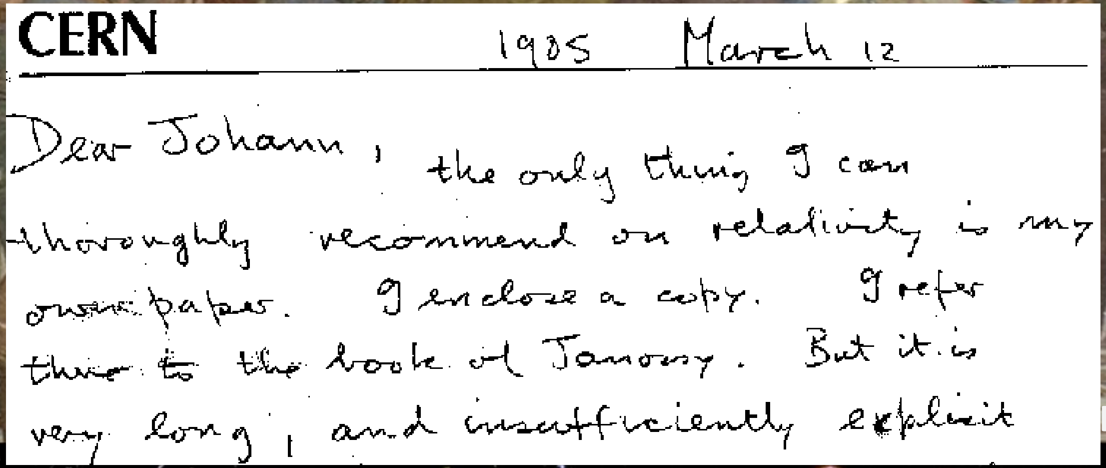
Lorentz-FitzGerald body contraction:

Is a passenger on a relativistic train aware they are “body contracted?”

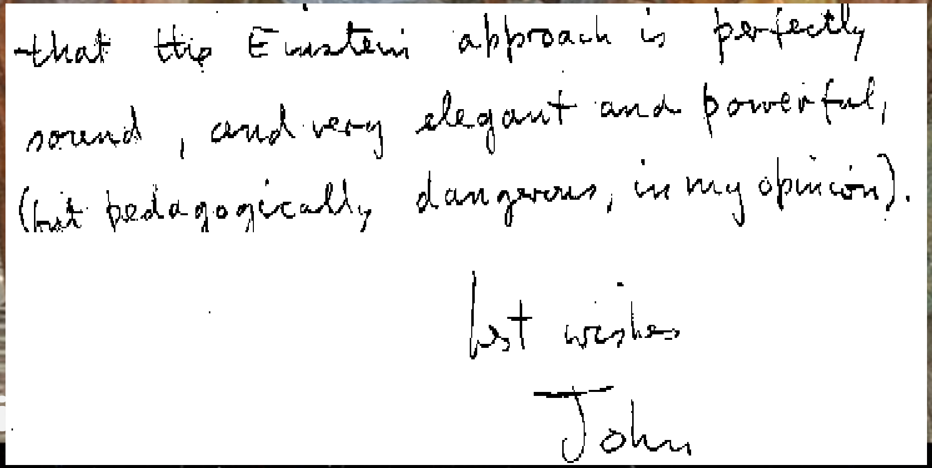
A. **Einstein, 1911:** No – there is no absolute reference frame in the Universe, they cannot know against what he or she contracts.

We know that the Big Bang reference frame defines speeds of all things in the Universe; is this relevant to understanding of SR?

J. S. Bell, 1976 (of “Bell inequality fame”) invokes Lorentz-Janossy reality point of view: Using **acceleration** the passenger transports from one inertial frame to another. This allows them to know and measure relative contraction.


CERN 1985 March 12

Dear Johann, the only thing I can thoroughly recommend on relativity is my own paper. I enclose a copy. I refer there to the book of Janossy. But it is very long, and insufficiently explicit


that the Einstein approach is perfectly sound, and very elegant and powerful, (but pedagogically dangerous, in my opinion).

best wishes

John

What is “real”?

Body contraction is real, but it is measurement process dependant.

Kinetic energy is real, but it is measurement process dependant.

Lorentz-FitzGerald body contraction:
Can we measure it?

Eur. Phys. J. A (2018) 54: 29
DOI 10.1140/epja/i2018-12370-4

Letter

THE EUROPEAN PHYSICAL JOURNAL A

Measurement of the Lorentz-FitzGerald body contraction

Johann Rafelski[a]

Dedicated to Walter Greiner; October 1935 – October 2016.

Published online: 20 February 2018

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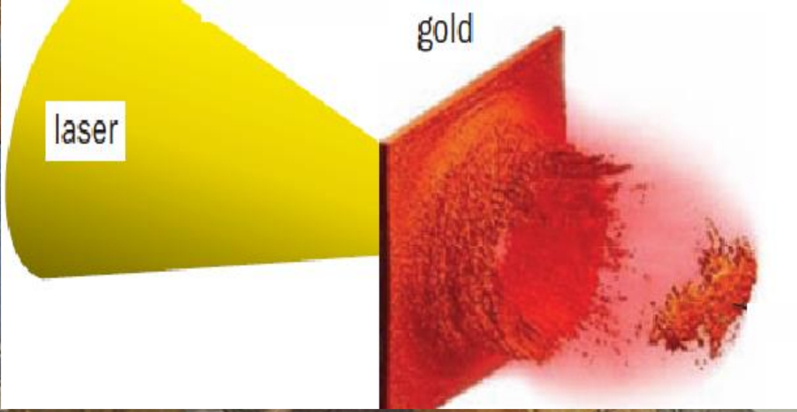
Abstract. A complete foundational discussion of acceleration in the context of Special Relativity (SR) is presented. Acceleration allows the measurement of a Lorentz-FitzGerald body contraction created. It is argued that in the back scattering of a probing laser beam from a relativistic flying electron cloud mirror generated by an ultra-intense laser pulse, a first measurement of a Lorentz-FitzGerald body contraction is feasible.

The "moving electron cloud mirror" is body compressed.

Body contraction experiment. — To accomplish our goal to build a laboratory-sized experiment we consider an ultra-intense ultra-short laser pulse shot at a thin (micron) foil. Such a pulse in its focal point can act as a micron-sized hammer pushing out of the foil an electron cloud accelerated to ultrarelativistic motion with a high value of Lorentz-factor γ_e. The emerging electron cloud compared to the original foil thickness will be Lorentz-FitzGerald compressed by γ_e.

A moving electron cloud acts as a relativistic mirror for a low intensity laser light bounce. The capability of the ultrarelativistic mirror to function depends on the electron cloud density; laser light can scatter coherently from a sufficiently high density cloud – what is low and high density is determined by comparing mean electron separation to the light wavelength.

two Lorentz transforms, first into the rest-frame of the mirror and upon reversal of the propagation direction of the light motion, transform back to the laboratory frame.



Time dilation is not reversible:
There is no twin “paradox” (See Langevin, 1911)

The only observer independent time quantity is the Lorentz invariant **proper time** of a body:

$$c^2\tau^2 = c^2t^2 - x^2 = c^2t'^2 - x'^2$$

Proper time of a body is meaningful and depends on $\boldsymbol{x}(\boldsymbol{t})$. A returning space traveller (who has accelerated) must have aged $\boldsymbol{\tau} < \boldsymbol{t}$.

If you reverse this, you introduce accelerated observers which are not yet incorporated in SR. Given proper time (which could be the lifespan of a particle) any two sets of values $\boldsymbol{t}$ and $\boldsymbol{x}$ are permissible associated with a specific Lorentz transform.

Time dilation requires acceleration which unlike velocity cannot be "removed" by choice of a suitable observer: velocity is "relative," acceleration is "absolute"

L' ÉVOLUTION DE L' ESPACE ET DU TEMPS 47

tions de la physique doivent conserver leur forme quand on passe de l' un à l' autre. Pour de tels systèmes tout se passe comme s' ils étaient immobiles par rapport à l' éther : une translation uniforme dans l' éther n' a pas de sens expérimental.

Mais il ne faut pas conclure pour cela, comme on l' a fait parfois prématurément, que la notion d' éther doit être abandonnée, que l' éther est inexistant, inaccessible à l' expérience. Seule une vitesse uniforme par rapport à lui ne peut être décelée, mais tout changement de vitesse, toute accélération a un sens absolu. En particulier c' est un point fondamental

L' ÉVOLUTION DE L' ESPACE ET DU TEMPS 49

aura moins vieilli entre son départ et son retour que si elle n' avait pas subi d' accélérations, que si elle était restée immobile par rapport à un système de référence en translation uniforme.

On peut dire encore qu' il suffit de s' agiter, de subir des accélérations pour vieillir moins vite ; nous allons voir dans un instant combien l' on peut espérer gagner de cette manière.

"...a uniform translation motion in the æther is not experimentally detectable... From this it should not be concluded, as has sometimes happened prematurely, that the æther must be abandoned having no physical reality since it cannot be experimentally probed. Only the uniform velocity relative to the æther cannot be detected, any change of velocity, that is, any acceleration, has an absolute meaning."

"Concluding, we can say it is sufficient to be set in motion, to experience acceleration in order to age less quickly."

- Langevin, Scientia X (1911)

Time dilation

The only observer independent time quantity is the Lorentz invariant **proper time $\boldsymbol{\tau}$** of a body:

$$c^2\tau^2 - 0 = c^2t^2 - x^2 = c^2t'^2 - x'^2$$

Proper time of a body ($\boldsymbol{x = 0}$) is meaningful. A returning traveller must have aged $\boldsymbol{\tau < t}$. For two planes going around the rotating Earth, one moves with speed added to rotation, while the other moves with speed subtracted from rotation. When they have made a full circle, they would have travelled different distances and recorded different passages of time.

Around-the-World Atomic Clocks: Predicted Relativistic Time Gains

Abstract. *During October 1971, four cesium beam atomic clocks were flown on regularly scheduled commercial jet flights around the world twice, once eastward and once westward, to test Einstein's theory of relativity with macroscopic clocks. From the actual flight paths of each trip, the theory predicts that the flying clocks, compared with reference clocks at the U.S. Naval Observatory, should have lost 40 ± 23 nanoseconds during the eastward trip, and should have gained 275 ± 21 nanoseconds during the westward trip. The observed time differences are presented in the report that follows this one.*

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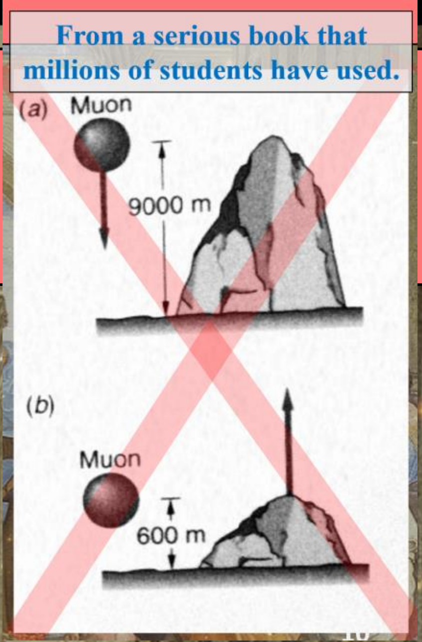
Unstable particle range



Imagine observing a muon in intergalactic empty space (no nearby mountains) so there is no LFG body contraction of anything. Using time dilation and the invariant spacetime interval, we can calculate the distance traveled.

$$c^2\tau^2 = c^2t^2 - x^2$$

Unstable particle proper time

$$x^2 = c^2t^2 - c^2\tau^2$$

Space-time interval

$$c^2\tau^2 = \left(1 - \frac{v^2}{c^2}\right)c^2t^2$$

Introducing speed of particle

$$\frac{E}{mc^2} \cong 14.15$$

$$v = \frac{x}{t}$$

$$x = \frac{\tau v}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{(2.197\ \mu\text{s})(0.9975c)}{\sqrt{1 - 0.9975^2}} \cong \mathbf{9.3\ km}$$

The muon travels at speed $\boldsymbol{v}$ for the distance $\boldsymbol{x}$ during its lifespan $\boldsymbol{\tau}$. **No Earth required!**

Relativistic Doppler effect (RDE):
No relation to time dilation

Time dilation of the source cannot be part of RDE since the relative speed with respect to the yet undetermined observer cannot be known at the time of light emission.

Einstein's 1905 paper works in the following way: The light wave carries to the observer information about the source allowing the determination of the RDE shift in frequency and wavelength and position aberration at the time of actual observation of the light signal.

$$\Phi = \Phi'$$

$$\omega t - \boldsymbol{k} \cdot \boldsymbol{x} = \omega' t' - \boldsymbol{k}' \cdot \boldsymbol{x}'$$

Use the Lorentz transformation for $\boldsymbol{x}'$ and t' to obtain Doppler effect including aberration.

As Einstein's argument is very terse and he presents without detailed calculation, it can be easily misunderstood. von Laue's SR book discussing RDE can also be misread.

How did the mix up between time dilation and Doppler effect happen?

~~Beobachter untersucht werden~~ — Durch Anwendung der in § 6 gefundenen Transformationsgleichungen für die elektrischen und magnetischen Kräfte und der in § 3 gefundenen Transformationsgleichungen für die Koordinaten und die Zeit erhalten wir unmittelbar:

$$X' = X_0 \sin \Phi', \qquad L' = L_0 \sin \Phi',$$

$$Y' = \beta\left(Y_0 - \frac{v}{V} N_0\right) \sin \Phi', \qquad M' = \beta\left(M_0 + \frac{v}{V} Z_0\right) \sin \Phi',$$

$$Z' = \beta\left(Z_0 + \frac{v}{V} M_0\right) \sin \Phi', \qquad N' = \beta\left(N_0 - \frac{v}{V} Y_0\right) \sin \Phi',$$

$$\Phi' = \omega'\left(\tau - \frac{a'\xi + b'\eta + c'\zeta}{V}\right),$$

wobei

$$\omega' = \omega\beta\left(1 - a\frac{v}{V}\right), \qquad a' = \frac{a - \frac{v}{V}}{1 - a\frac{v}{V}},$$

$$b' = \frac{b}{\beta\left(1 - a\frac{v}{V}\right)}, \qquad c' = \frac{c}{\beta\left(1 - a\frac{v}{V}\right)}$$

gesetzt ist.

Aus der Gleichung für ω' folgt: Ist ein Beobachter relativ zu einer unendlich fernen Lichtquelle von der Frequenz ν mit der Geschwindigkeit v derart bewegt, daß die Verbindungslinie „Lichtquelle–Beobachter" mit der auf ein relativ zur Lichtquelle ruhendes Koordinatensystem bezogenen Geschwindigkeit des Beobachters den Winkel φ bildet, so ist die von dem Beobachter wahrgenommene Frequenz ν' des Lichtes durch die Gleichung gegeben:

$$\nu' = \nu \frac{1 - \cos\varphi \frac{v}{V}}{\sqrt{1 - \left(\frac{v}{V}\right)^2}}.$$

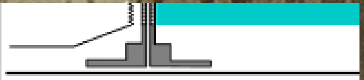
Dies ist das Doppelersche Prinzip für beliebige Geschwindig-

Ives and Stilwell in 1938 measure (transverse) Doppler shift claiming they measure time dilation.

Ives–Stilwell experiment

From Wikipedia, the free encyclopedia

https://en.wikipedia.org/wiki/Ives–Stilwell_experiment



Ives–Stilwell experiment (1938). "Ca... mostly H_2^+ and H_3^+ ions) were acce...

The **Ives–Stilwell experiment** tested the contribution of relativistic time dilation to the Doppler shift of light.[1][2] The result was in agreement with the formula for the transverse Doppler effect and was the first direct, quantitative confirmation of the time dilation factor.

Resnick around 1960 learns using what appears to be garbled translation of von Laue's SR book and relies on the language of Ives-Stilwell. **This is copied in most English language books and is today found all over the internet.**

Further reading on RDE

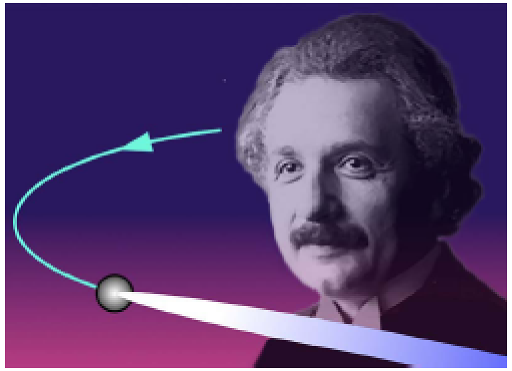
JOURNAL OF SYNCHROTRON RADIATION

ISSN 1600-5775

Received 4 April 2017
Accepted 24 May 2017

Edited by M. Eriksson, Lund University, Sweden

Keywords: special relativity; Doppler; time dilation; Lorentz transformation.



OPEN ACCESS

The relativistic foundations of synchrotron radiation

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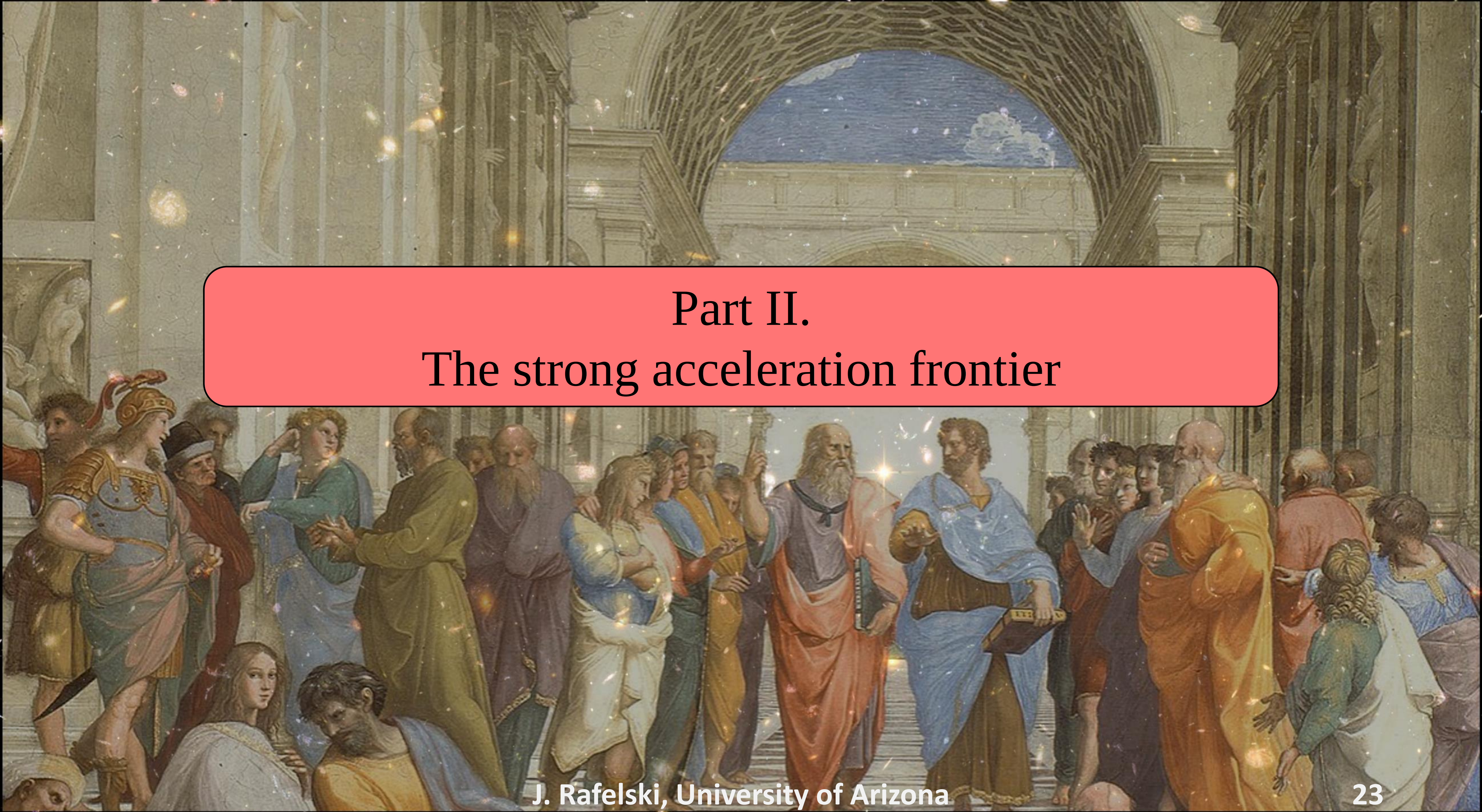
Special relativity (SR) determines the properties of synchrotron radiation, but the corresponding mechanisms are frequently misunderstood. Time dilation is often invoked among the causes, whereas its role would violate the principles of SR. Here it is shown that the correct explanation of the synchrotron radiation properties is provided by a combination of the Doppler shift, not dependent on time dilation effects, contrary to a common belief, and of the Lorentz transformation into the particle reference frame of the electromagnetic field of the emission-inducing device, also with no contribution from time dilation. Concluding, the reader is reminded that much, if not all, of our argument has been available since the inception of SR, a research discipline of its own standing.



Part II.
The strong acceleration frontier


Completing EM interactions: Covariant classical radiation reaction

$$\tau_0 = \frac{2}{3}\frac{e^2}{4\pi m c^3}$$

$$P^{\mu\nu} = g^{\mu\nu} - \frac{u^\mu u^\nu}{u^2}$$

Principle models:

$$m a^\mu = \frac{e}{c} F^{\mu\nu} u_\nu + m\tau_0 \left(\frac{d a^\mu}{d\tau} + \frac{a_\nu a^\nu}{c^2} u^\mu \right)$$

Lorentz-Abraham-Dirac (LAD)

As far as Jackson text goes

$$m a^\mu = \frac{e}{c} F^{\mu\nu} u_\nu + e\tau_0 \left(u \cdot \partial F^{\mu\nu} u_\nu + \frac{e}{m} P^{\mu\nu} F_{\nu\alpha} F^{\alpha\beta} u_\beta \right)$$

Landau-Lifshitz (LL)

As far as LL text goes

$$m a^\mu = \frac{e}{c} F^{\mu\nu} u_\nu + \tau_0 P^\mu_\nu \frac{d}{d\tau} \left(\frac{e}{c} F^{\nu\alpha} u_\alpha \right)$$

Eliezer-Ford-O'Connell (EFO)

The Cinderella of RR?

W. Price, M. Formanek, and J. Rafelski. "Radiation reaction and limiting acceleration". PRD 105 (2022)

P. A. M. Dirac, "Classical theory of radiating electrons," Proc. R. Soc. A 167, 148 (1938)

L. D. Landau and E. M. Lifshitz, The Classical Theory of Fields, 2ed, London, England: Pergamon (1962)

S. E. Gralla, A. I. Harte, R. M. Wald. "A Rigorous Derivation of Electromagnetic Self-force." Rev. D80, 024031(2009)

Distinct features of radiation reaction models

LAD

- Requires self-interaction
- Unphysical runaway solutions
- Computationally impossible

Kinematic variables only
$\boldsymbol{a^\mu, \dot{a}^\mu}$

LL

- Equivalent to LAD in perturbative limit
- Useless for strong accelerations

Field variables only
$\boldsymbol{F^{\mu\nu}, \dot{F}^{\mu\nu}}$

EFO

- Maximum limiting acceleration.
- Equivalent to LL for weak acceleration.

Kinematic and Fields
$\boldsymbol{a^\mu, \dot{F}^{\mu\nu}}$

Name	Covariant equation	Year
Lorentz-Abraham-Dirac (LAD)	$ma^\mu = \mathcal{F}^\mu + \tau_0 P^\mu_\nu \frac{d}{d\tau}(ma^\nu)$	1938
Eliezer-Ford-O'Connell (EFO)	$ma^\mu = \mathcal{F}^\mu + \tau_0 P^\mu_\nu \frac{d}{d\tau}(eF^{\nu\alpha}u_\alpha)$	1948, 1991
Landau-Lifshitz (LL)	$ma^\mu = \mathcal{F}^\mu + \tau_0(e\frac{d}{d\tau}(F^{\mu\nu})u_\nu + \frac{e^2}{m}P^\mu_\nu F^{\nu\alpha}F_{\alpha\beta}u^\beta)$	1962
Mo-Papas (MP)	$ma^\mu = \mathcal{F}^\mu + e\tau_0 P^\mu_\nu F^{\nu\alpha}a_\alpha$	1971

W. Price, M. Formanek, and J. Rafelski. "Radiation reaction and limiting acceleration". PRD 105 (2022)
P. A. M. Dirac, "Classical theory of radiating electrons," Proc. R. Soc. A 167, 148 (1938)
L. D. Landau and E. M. Lifshitz, The Classical Theory of Fields, 2ed, London, England: Pergamon (1962)
S. E. Gralla, A. I. Harte, R. M. Wald. "A Rigorous Derivation of Electromagnetic Self-force." Rev. D80, 024031(2009)

Radiation reaction and limiting acceleration

PHYSICAL REVIEW D **105,** 016024 (2022)

Radiation reaction and limiting acceleration

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(Received 9 December 2021; accepted 7 January 2022; published 26 January 2022)

We investigate the strong acceleration properties of the radiation reaction force and identify a new and promising limiting acceleration feature in the Eliezer-Ford-O'Connell model; in the strong field regime, for many field configurations, we find an upper limit to acceleration resulting in a bound to the rate of radiation emission. If this model applies, strongly accelerated particles are losing energy at a much slower pace than predicted by the usual radiation reaction benchmark, the Landau-Lifshitz equation, which certainly cannot be used in this regime. We explore examples involving various "constant" electromagnetic field configurations and study particle motion in a light plane wave as well as in a material medium.

DOI: 10.1103/PhysRevD.105.016024


Radiation reaction and limiting acceleration

$$\tau_0 = \frac{2}{3}\frac{e^2}{4\pi mc^3}$$

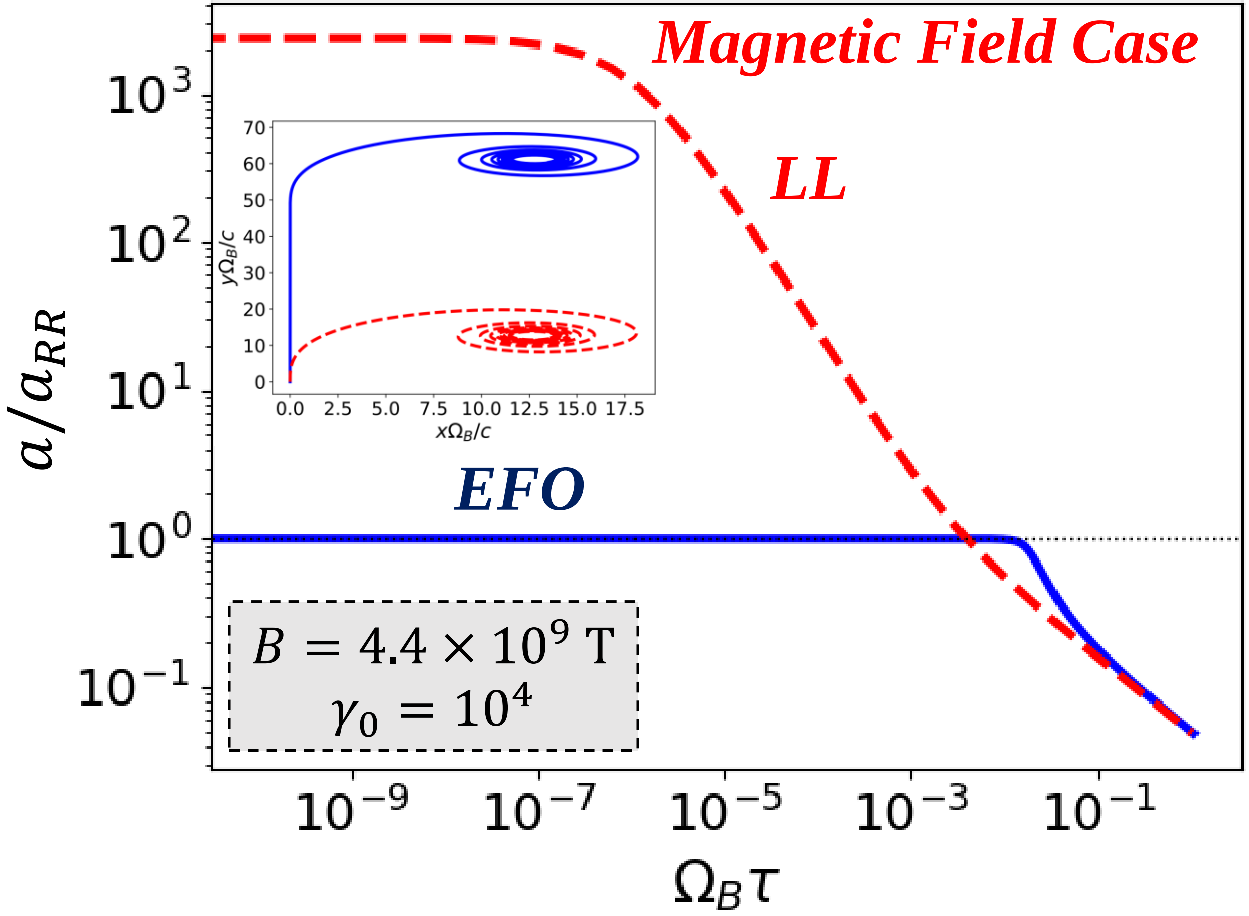
Eliezer-Ford-O'Connell (EFO) in homogenous fields

$$a^2 = -a_{LF}^2 \frac{1 + \tau_0^2 \frac{e^4}{m^4}\frac{c^2\mathcal{P}^2}{|a_{LF}^2|}}{1 + \tau_0^2\left(\frac{e^2}{m^2}2\mathcal{S} + \frac{|a_{LF}^2|}{c^2}\right)}$$

$$\lim_{\gamma\to\infty} a^2 \to -\frac{c^2}{\tau_0^2}$$

$$|a_{RR}| = \frac{c}{\tau_0}$$

Limiting acceleration: A common feature with Born-Infeld EM theory



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$$a_{LF}^\mu = \frac{e}{m}F^{\mu\nu}u_\nu$$

$$\Omega_B = \frac{eB}{m}$$

Omitting problematic Schott term

$$m\tau_0 \ddot{u}^\mu$$

Path warping: The new idea for radiation reaction

Start with point external force + Larmor term

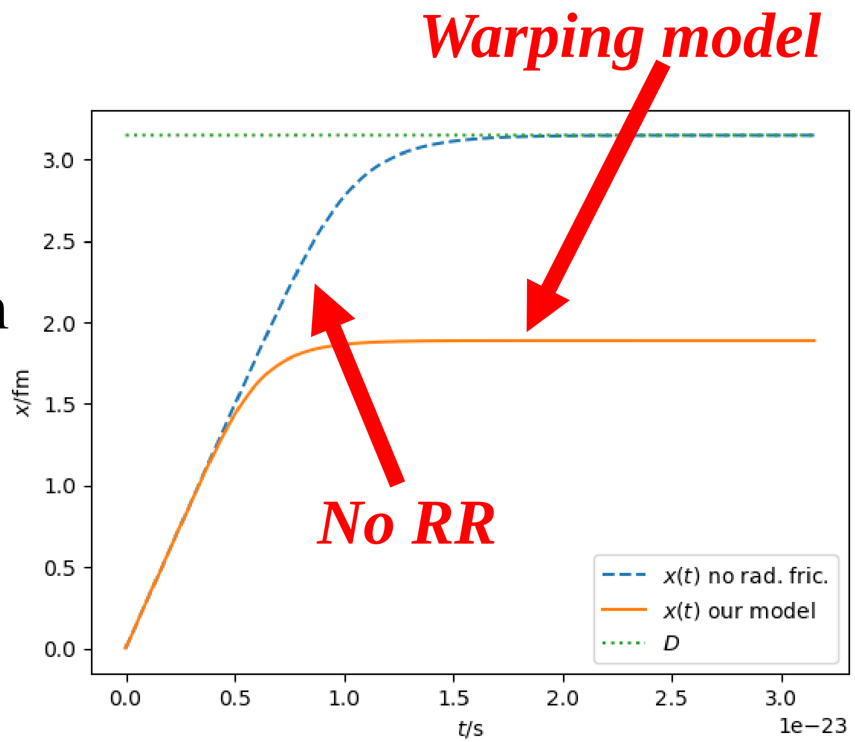
$$m\dot{u}^\mu = f^\mu + m\tau_0 \frac{\dot{u}^2}{c^2} u^\mu$$

Introduce "path warping" for particles with medium friction

$$u \cdot \dot{u} = \tau_0 \dot{u}^2 \neq 0 \quad \leftrightarrow \quad u^2 = w_{\mu\nu} u^\mu u^\nu = c^2$$

Path warping along world-line of particle

$$u \cdot \dot{u} = -\frac{1}{2}\frac{dw_{\mu\nu}}{d\tau} u^\mu u^\nu \neq 0$$



Unique particle stopping distance predictions versus other models.

Applications for quark jet quenching in QGP

Outlook: We hope to connect an action principle with the idea of warping in vacuum.

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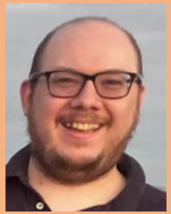


More need to complete EM interactions: Unified covariant classical magnetic dipole interaction

Electric energy: $E_{el} = ecA^0$

Magnetic energy: $E_{mag} = d_m cB^0$ → $\boldsymbol{\mu} = (d_m c)\boldsymbol{S}$

Magnetic dipole charge

A covariant magnetic potential B^μ can be introduced

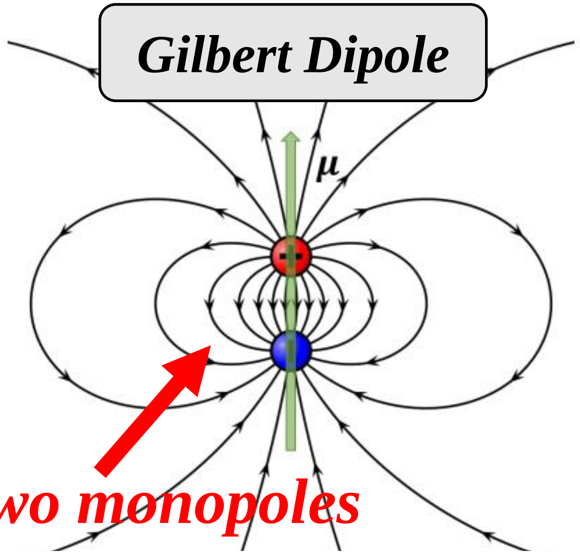
$$B_\mu \equiv F^*_{\mu\nu} s^\nu = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}F^{\alpha\beta}s^\nu$$ → $$G^{\mu\nu} \equiv \partial^\mu B^\nu - \partial^\nu B^\mu$$

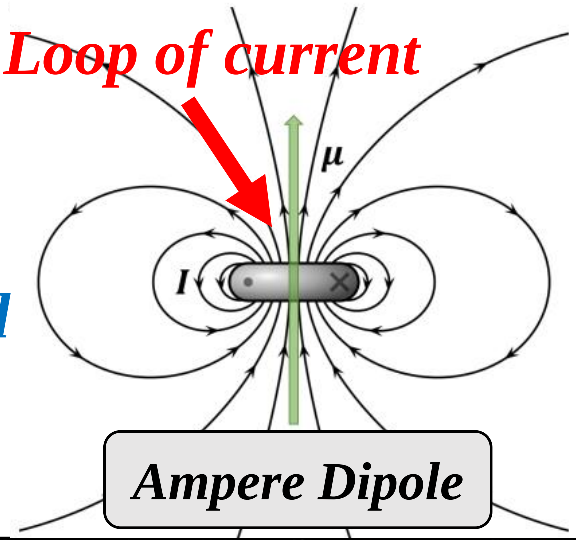
Define a Force Field Tensor

Point particle classical Lagrangian

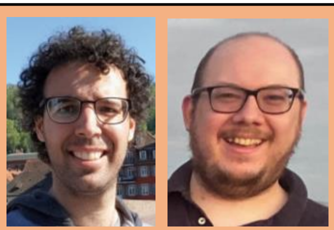
$$L = mc\sqrt{u^2} + eA \cdot u + d_m B \cdot u$$

Covariant description contains both Gilbert and Ampere dipole structure





J. Rafelski, M. Formanek, and A. Steinmetz. "Relativistic dynamics of point magnetic moment." EPJC 78.1 (2018): 1-12.



Completing EM interactions: Unified covariant classical magnetic dipole interaction

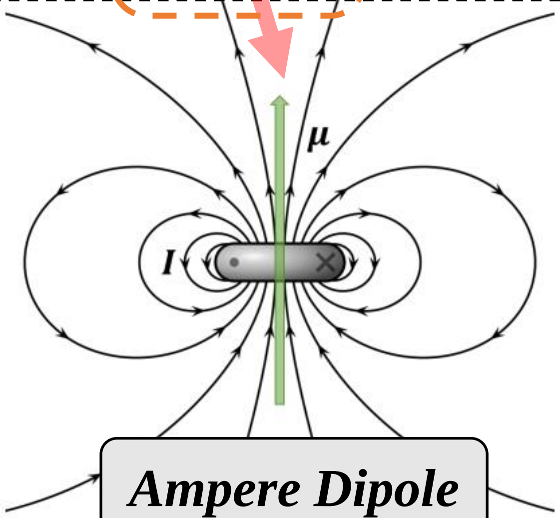
The equations of motion for the above are then

$$\dot{u}^{\mu} = \frac{e}{m} F^{\mu\nu} u_{\nu} - \frac{d_m}{m} s \cdot \partial (F^{*\mu\nu}) u_{\nu} - \frac{d_m}{m} \mu_0 \epsilon^{\gamma\alpha\beta\mu} j_{\gamma} u_{\alpha} s_{\beta}$$

We can solve this, not just academic!
(Example in supplementary material)

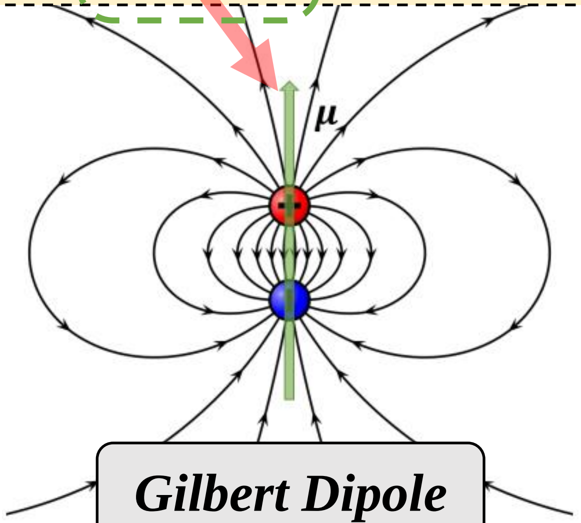
Comoving Frame (CF)

$$\boldsymbol{F}\Big|_{\mathrm{CF}} = e\boldsymbol{E} + \boldsymbol{\nabla}(\boldsymbol{\mu} \cdot \boldsymbol{B}) - \boldsymbol{\mu} \times \frac{\partial \boldsymbol{E}}{\partial t} = e\boldsymbol{E} + (\boldsymbol{\mu} \cdot \boldsymbol{\nabla})\boldsymbol{B} + \mu_0 \boldsymbol{\mu} \times \boldsymbol{j}$$



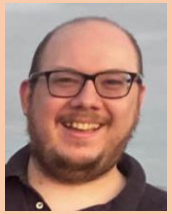
Ampere Dipole

OR



Gilbert Dipole

J. Rafelski, M. Formanek, and A. Steinmetz. "Relativistic dynamics of point magnetic moment." EPJC 78.1 (2018): 1-12.



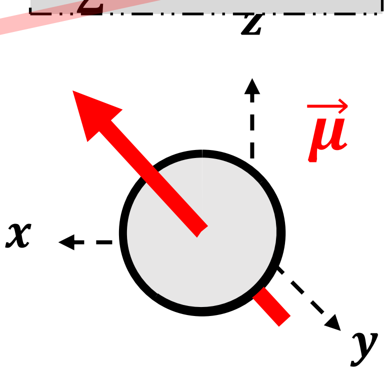
Quantum magnetic dipoles: Diverse forms of quantum equations

$$\boldsymbol{\mu} \leftarrow \frac{g}{2}\frac{e\hbar}{2m}$$

$$\frac{g}{2} = 1 + a$$

Non-relativistic magnetic dipole has the Hamiltonian:

$$\hat{H}_{Mag.} = -\vec{\boldsymbol{\mu}} \cdot \vec{\boldsymbol{B}}$$



Relativistic magnetic dipoles have a diversity of models:

$$\left(\gamma \cdot (i\hbar\partial - eA) - mc - \left(\boldsymbol{\mu} - \frac{e\hbar}{2m}\right)\frac{1}{2c}\sigma_{\alpha\beta}F^{\alpha\beta}\right)\psi = 0$$ **Dirac-Pauli (DP)**

$$\left((i\hbar\partial - eA)^2 - m^2c^2 - \boldsymbol{\mu} m\sigma_{\alpha\beta}F^{\alpha\beta}\right)\psi = 0$$ **Klein-Gordon-Pauli (KGP)**

$$\left((i\hbar\partial - eA)^2 - \tilde{m}^2c^2\right)\psi = 0$$ **"Improved" Klein-Gordon-Pauli (IKGP)**

$$\tilde{m}c = mc + \boldsymbol{\mu}\frac{1}{2c}\sigma_{\alpha\beta}F^{\alpha\beta} \rightarrow \tilde{m}^2c^2 = m^2c^2 + \boldsymbol{\mu} m\sigma_{\alpha\beta}F^{\alpha\beta} + \boldsymbol{\mu}^2\frac{1}{4c^2}\left(\sigma_{\alpha\beta}F^{\alpha\beta}\right)^2$$

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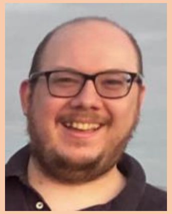
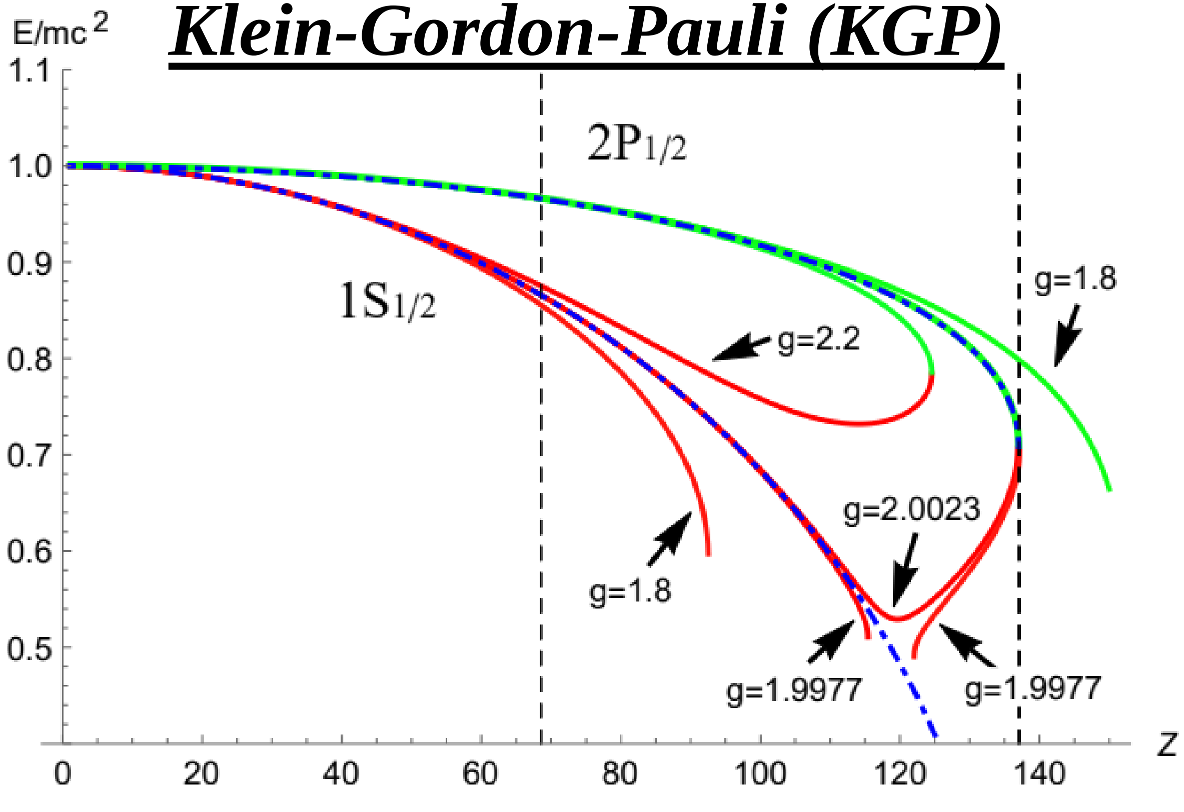
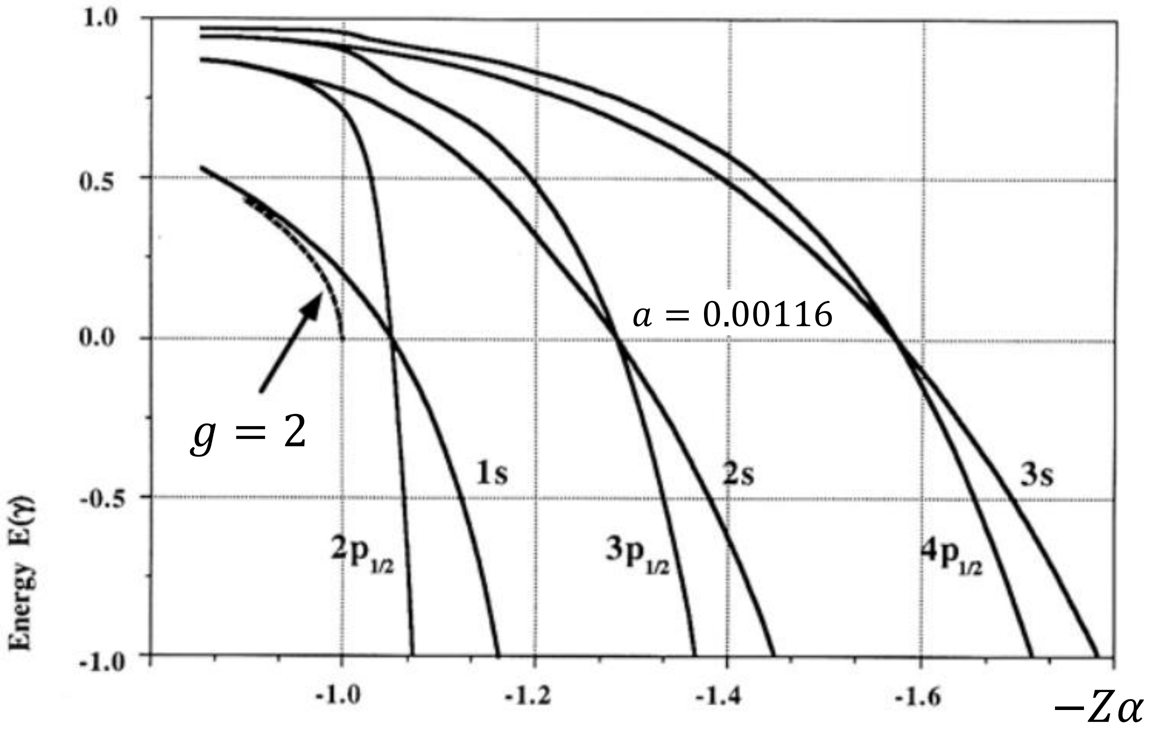
Strong Coulomb field eigen-energies



Klein-Gordon-Pauli (KGP)



Dirac-Pauli (DP)



A. Steinmetz, M. Formanek, and J. Rafelski. "Magnetic dipole moment in relativistic quantum mechanics." EPJA 55.3 (2019): 1-17.

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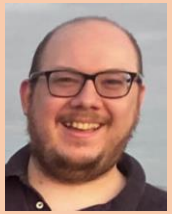
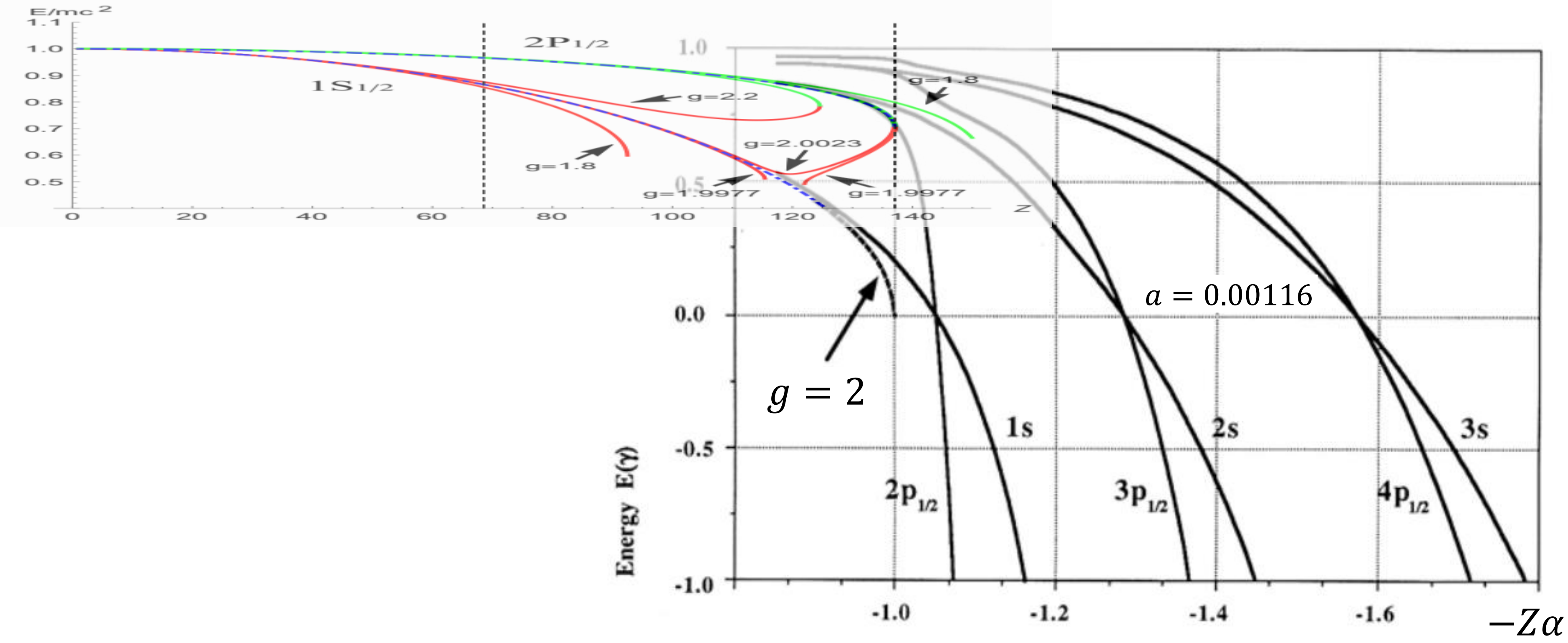


Strong Coulomb field eigen-energies

KGP and DP Spectrum with Same Scaling



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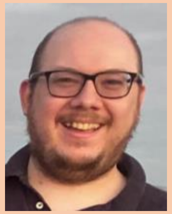
B. Thaller. The Dirac equation. Springer Science & Business Media, 2013.

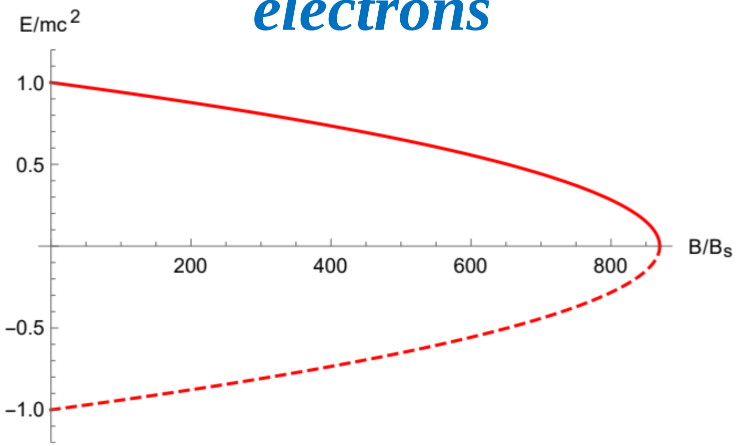


Constant magnetic field eigen-energies

$$\boldsymbol{B_S} \equiv \frac{m^2c^2}{e\hbar} = \begin{cases} 4.41 \times 10^9 \text{ T (electrons)} \\ 1.49 \times 10^{16} \text{ T (protons)} \end{cases}$$

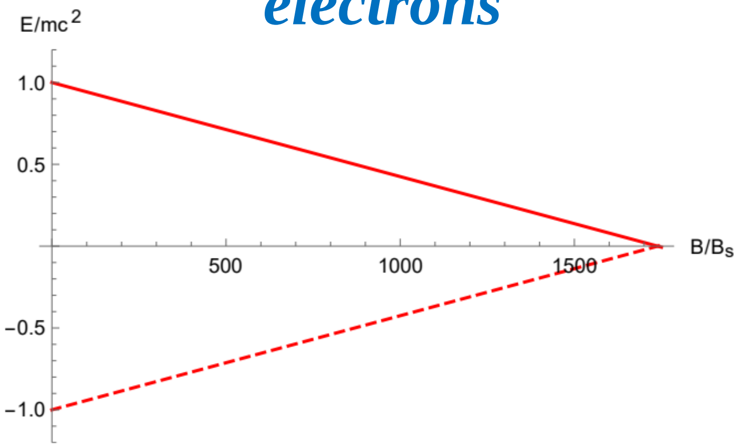
Klein-Gordon-Pauli (KGP)

electrons



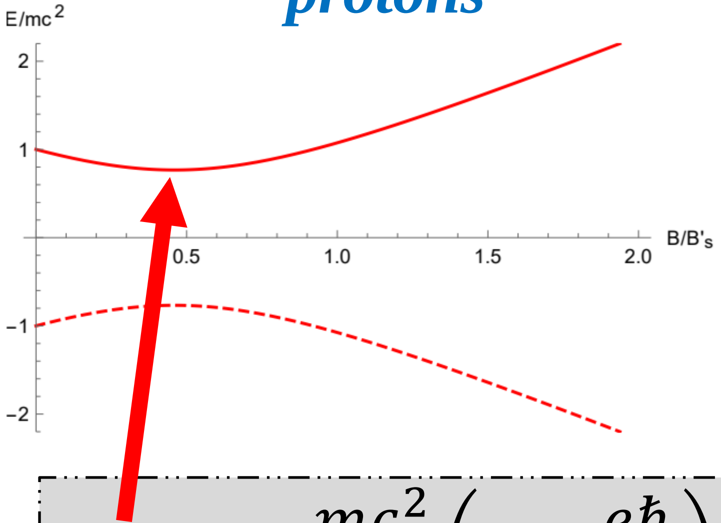
Dirac-Pauli (DP)

electrons



"Improved" Klein-Gordon-Pauli (IKGP)

protons



$$\boldsymbol{B_{min}} \equiv \frac{mc^2}{\mu}\left(\mu - \frac{e\hbar}{2m}\right)$$

Expect grossly different properties in magnetars.

A. Steinmetz, M. Formanek, and J. Rafelski. "Magnetic dipole moment in relativistic quantum mechanics." EPJA 55.3 (2019): 1-17.

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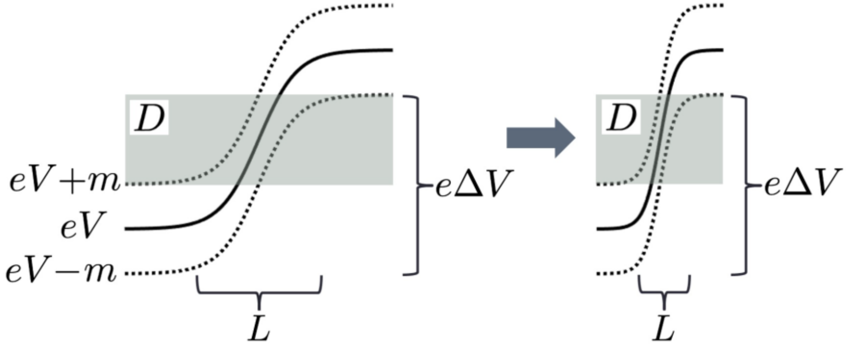
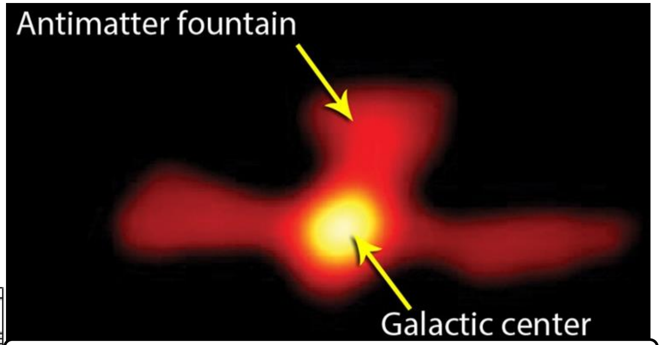
Sauter potential step

$$V_z = \frac{\mathcal{E}_0 L}{2}\tanh\left[\frac{2z}{L}\right]$$

Transition in pair production from Euler-Heisenberg to Klein paradox limit





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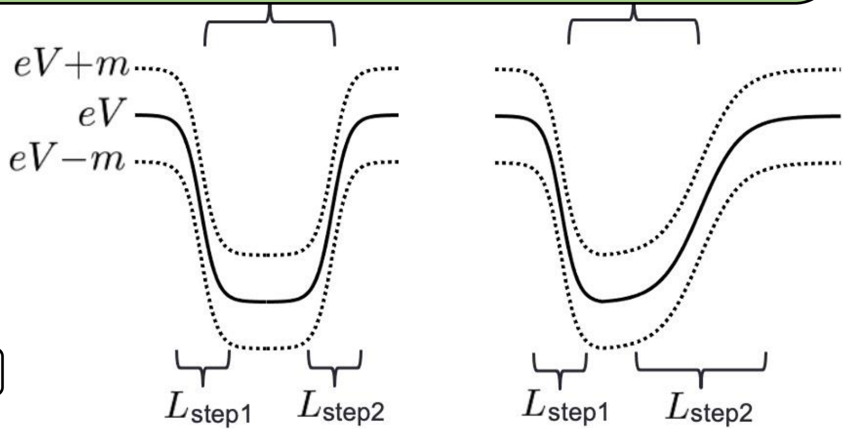

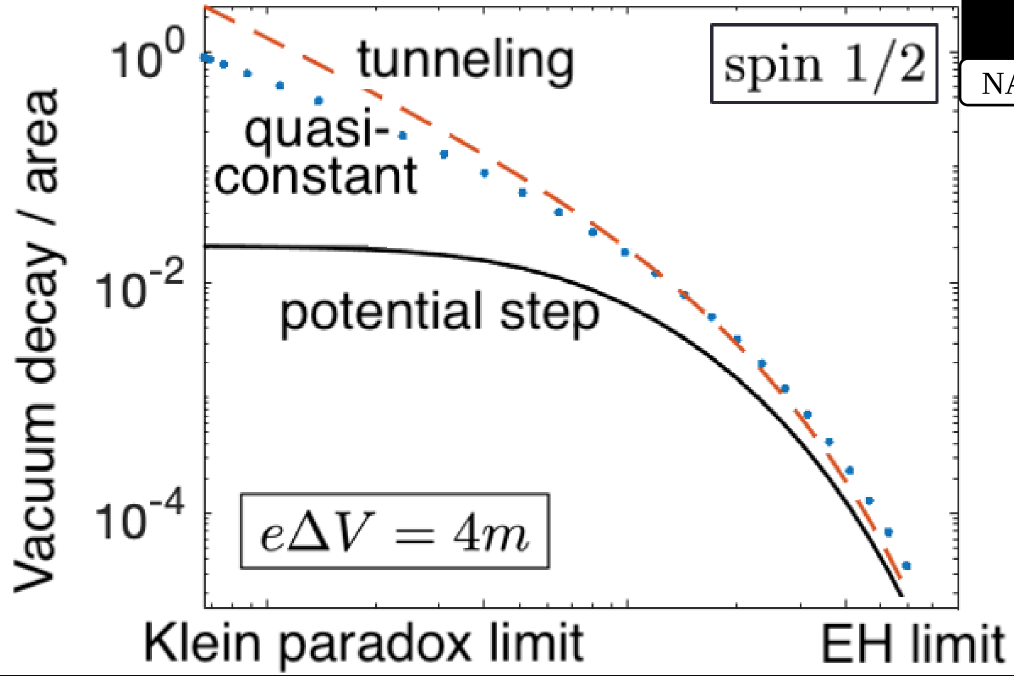

Step to well

Single potential step or two steps forming a well:

- Finite pair production per unit area versus the diverging rate per volume

Two steps forming a well required for:

- A good definition of vacuum
- Pair production highly sensitive to the shape of the well

Next Slide

As step becomes steep, the forces become very large inducing radiation effects.

S. Evans and J. Rafelski. “Particle production at a finite potential step: Transition from Euler-Heisenberg to Klein paradox.” (2021) [arXiv:2108.12959]

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Role of magnetic moment periodicity

Eur. Phys. J. A (2021) 57:341
https://doi.org/10.1140/epja/s10050-021-00654-x

THE EUROPEAN PHYSICAL JOURNAL A

Regular Article - Theoretical Physics

Particle production at a finite potential step: transition from Euler–Heisenberg to Klein paradox

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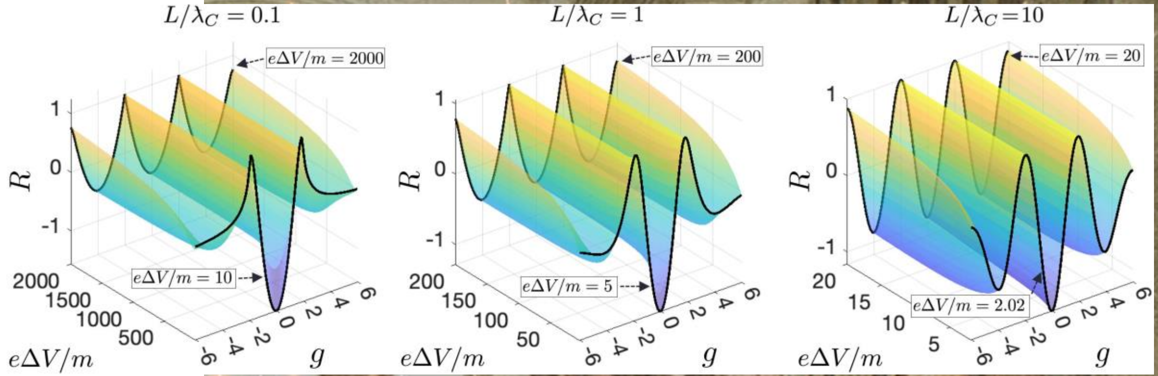
Received: 8 November 2021 / Accepted: 18 December 2021 / Published online: 31 December 2021

Communicated by Tamas Biro

Abstract Spontaneous pair production for spin-1/2 and spin-0 particles is explored in a quantitative manner for a static tanh-Sauter potential step (SS), evaluating the imaginary part of the effective action. We provide finite-valued per unit-surface results, including the exact sharp-edge Klein paradox (KP) limit, which is the upper bound to pair production. At the vacuum instability threshold the spin-0 particle production can surpass that for the spin-1/2 rate. Presenting the effect of two opposite sign Sauter potential steps creating a well we show that spin-0 pair production, contrary to the case of spin-1/2, requires a smoothly sloped wall.



Emergence of periodic in magnetic moment effective QED action

Stefan Evans[a,*], Johann Rafelski[a]

[a]*Department of Physics, The University of Arizona, Tucson, AZ 85721, USA*

Abstract

We evaluate for the inhomogeneous static electric Sauter step potential the imaginary part of the emerging homogeneous in electric field effective Euler-Heisenberg-Schwinger action sourced by vacuum fluctuations of a charged particle with magnetic moment of arbitrary strength. The result is convergent for all values of gyromagnetic ratio g, periodic in g, with a cusp at $g = 2$. We consider the relation to the QED beta-function which is also found to be periodic in g. We confirm presence of asymptotic freedom conditions using this novel method and document a wider range of g-values for which asymptotic freedom is present.

Magnetic moment and major modification of pair production

KGP introduces corrections into Euler-Heisenberg (**EH**) action:

- Pair production modification due to periodicity of g.

$$L_{EH} = -\frac{1}{8\pi^2}\int_{\delta}^{\infty}\frac{ds}{s^3}e^{-m_e^2 s}\left(\frac{abe^2s^2\cos\left[\frac{g}{2}eas\right]\cosh\left[\frac{g}{2}ebs\right]}{\sin[eas]\sinh[ebs]}-1\right)$$

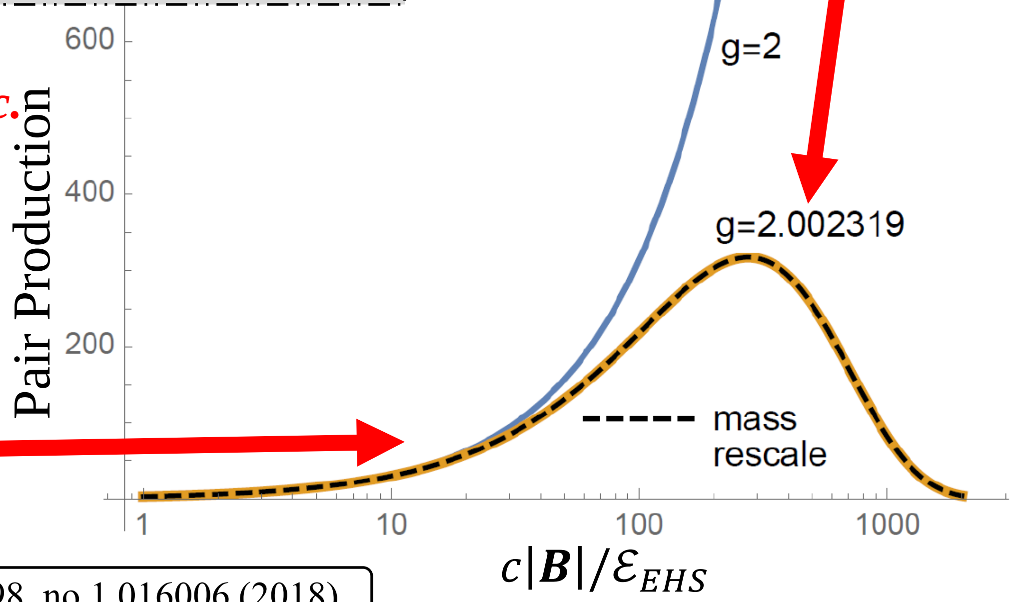
Origin of electron mass? Higgs and electromagnetic.

$$\mathcal{E}_{EHS} = \frac{m_e^2 c^3}{e\hbar} = 1.323\times 10^{18}\,\frac{\text{V}}{\text{m}}$$

$$a^2 - b^2 = E^2 - B\text{^}2$$
$$a^2 b^2 = (E\cdot B)^2$$

$$m_{rescale}^2 c^2 = m_e^2 c^2 + \left|\frac{g}{2}-1\right| e\hbar B$$

Modification to pair production



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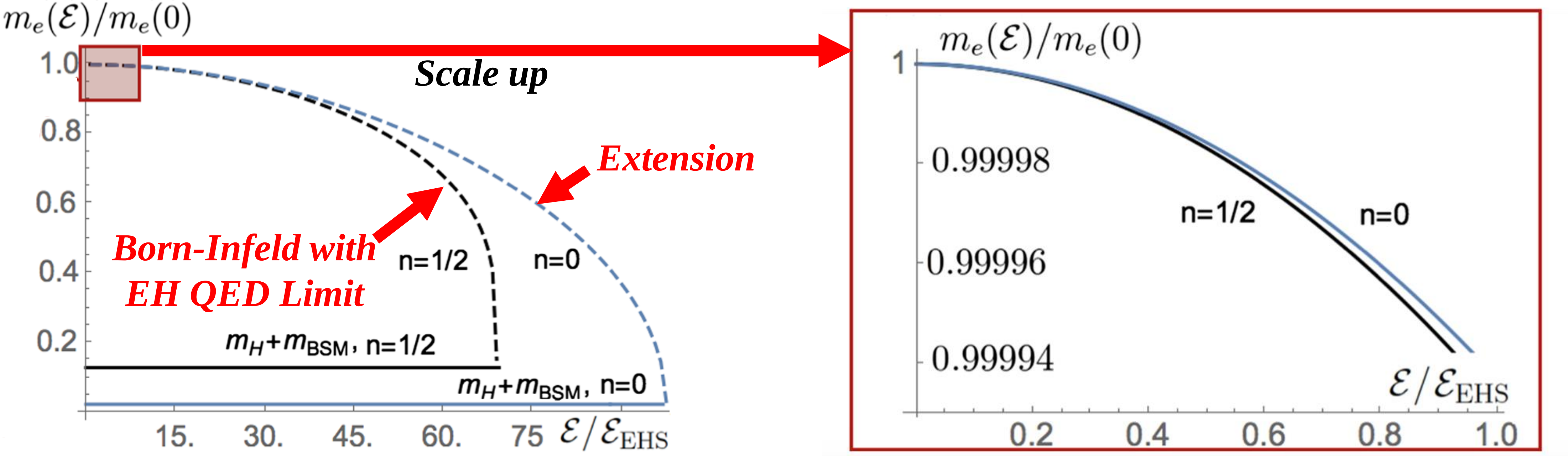
Strong fields as probes of the origin of electron mass

Origin of mass:

- EM and non-EM (Higgs+BSM) mass components
- EM mass melting in external fields
- Self-consistent feedback with nonlinear EM action

Using Born-Infeld model of the electron.



EM mass content controlled by model parameter $\boldsymbol{n}$

S. Evans and J. Rafelski. "Electron electromagnetic-mass melting in strong fields." Phys. Rev. D 102, 036014 (2020)

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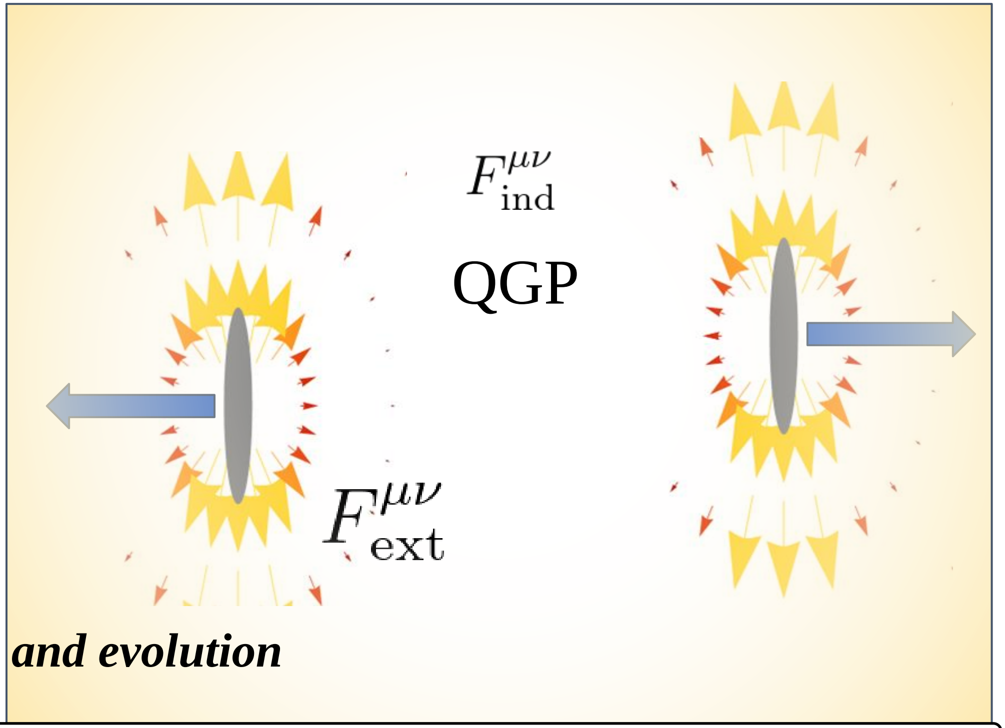
I. Birula. "Nonlinear Electrodynamics: Variations On A Theme By Born And Infeld." In: B. Jancewicz, J. Lukierski: Quantum Theory Of Particles and Fields, World Scientific (1983)

Strong acceleration probed in heavy-ion collisions

Ultra-strong electromagnetic fields in heavy-ion collisions:

- Combined self consistent ion and QGP electromagnetic fields
- Probing short lived systems, EM pair production and QGP plasma

We model the QGP as an infinite plasma.



Qualitative illustrations of QGP formation and evolution

C. Grayson, M. Formanek, B. Müller, J. Rafelski, "EM Polarization of Quark-Gluon Plasma" In preparation. (2021)

M. Formanek, C. Grayson, J. Rafelski, B. Müller, "Current-Conserving Relativistic Linear Response for Collisional Plasmas" Annals of Physics 434 (2021) doi:10.1016/j.aop.2021.168605 [arXiv:2105.07897]

K. Tuchin, "Particle production in strong electromagnetic fields in relativistic heavy-ion collisions." Advances in High Energy Physics (2013)


Quark-gluon plasma (QGP) *probed in heavy-ion collisions*

Scattering damping: κ
Medium 4-velocity: u
Distribution function: f_{eq}

$$\frac{m_\pi^2 c^2}{e\hbar} \approx 3.1 \times 10^{14}\ \mathrm{T}$$

The induced EM fields $F^{\mu\nu}_{ind}$ generated by QGP can be modelled using the Vlasov-Boltzmann equation with scattering term.

$$(p \cdot \partial) f(x,p) + qF^{\mu\nu}_{ext} p_\nu \left(\frac{\partial f(x,p)}{\partial p^\mu} \right) = \kappa (p \cdot u) \left(f_{eq}(p) \frac{n(x)}{n_{eq}} - f(x,p) \right)$$

The induced 4-current $J^\mu_{ind}(k)$, in Fourier modes, is then

$$\tilde{J}^\mu_{ind}(k) = 2N_c \int \frac{d^4p}{(2\pi)^4} 4\pi \delta_+(p^2 - m^2) p^\mu \sum_{u,d,s} q_f \left(\tilde{f}_f(k,p) - \tilde{f}_{\bar{f}}(k,p) \right)$$

The polarization can then be identified.

$$j_i = \sigma_{ij} E_j$$

Relativistic Ohm's Law in QGP

$$\tilde{J}^\mu_{ind}(k) = \Pi^\mu_\nu \tilde{A}^\nu(k)$$

Strong electromagnetic polarization modifies QGP

C. Grayson, M. Formanek, B. Müller, J. Rafelski, "EM Polarization of Quark-Gluon Plasma" In preparation. (2021)
M. Formanek, C. Grayson, J. Rafelski, B. Müller, "Current-Conserving Relativistic Linear Response for Collisional Plasmas" Annals of Physics 434 (2021) doi:10.1016/j.aop.2021.168605 [arXiv:2105.07897]
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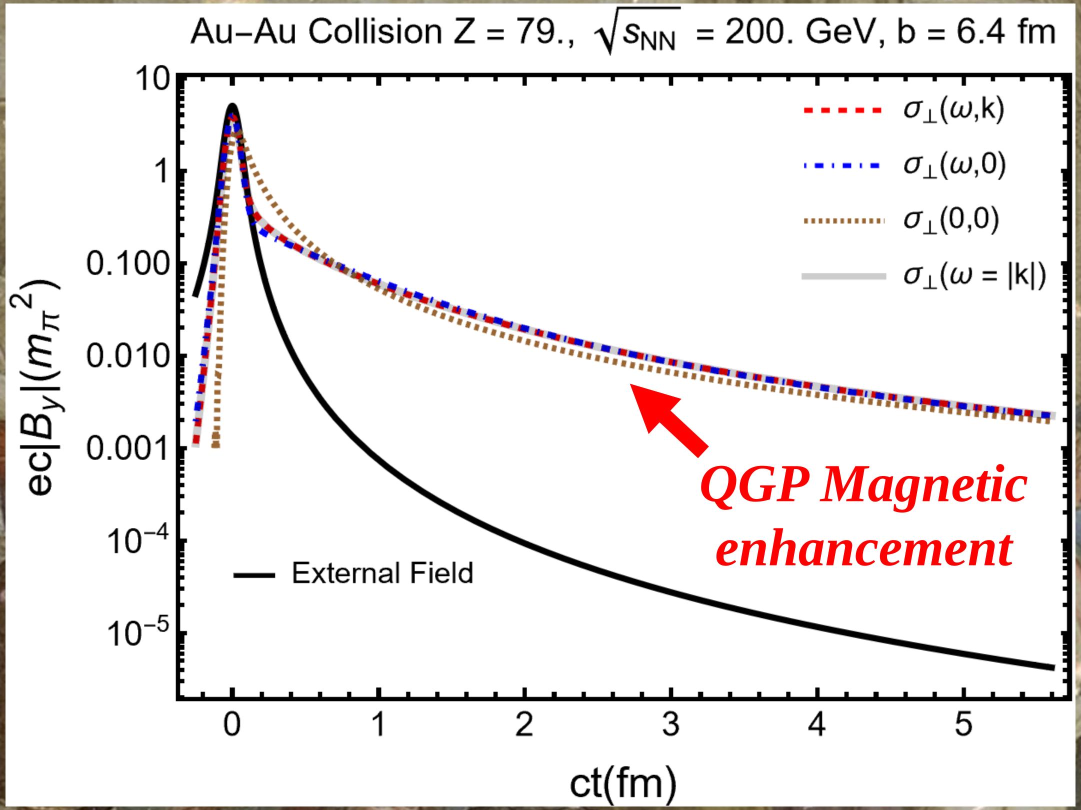
QGP magnetic enhancement: Magnetic field at the geometric origin of particle collision



The magnetic field spikes and then drops at the origin of the collision as a function of time.

$$\sigma_\perp = \frac{i m_D^2}{4\omega}\left(\frac{\kappa^2}{\omega^2}\xi \ln\,\xi + \frac{i\kappa}{\omega}(\xi+1)\right)$$

Transverse Conductivity

$$\xi = 1 - \frac{2i\omega}{\kappa}$$

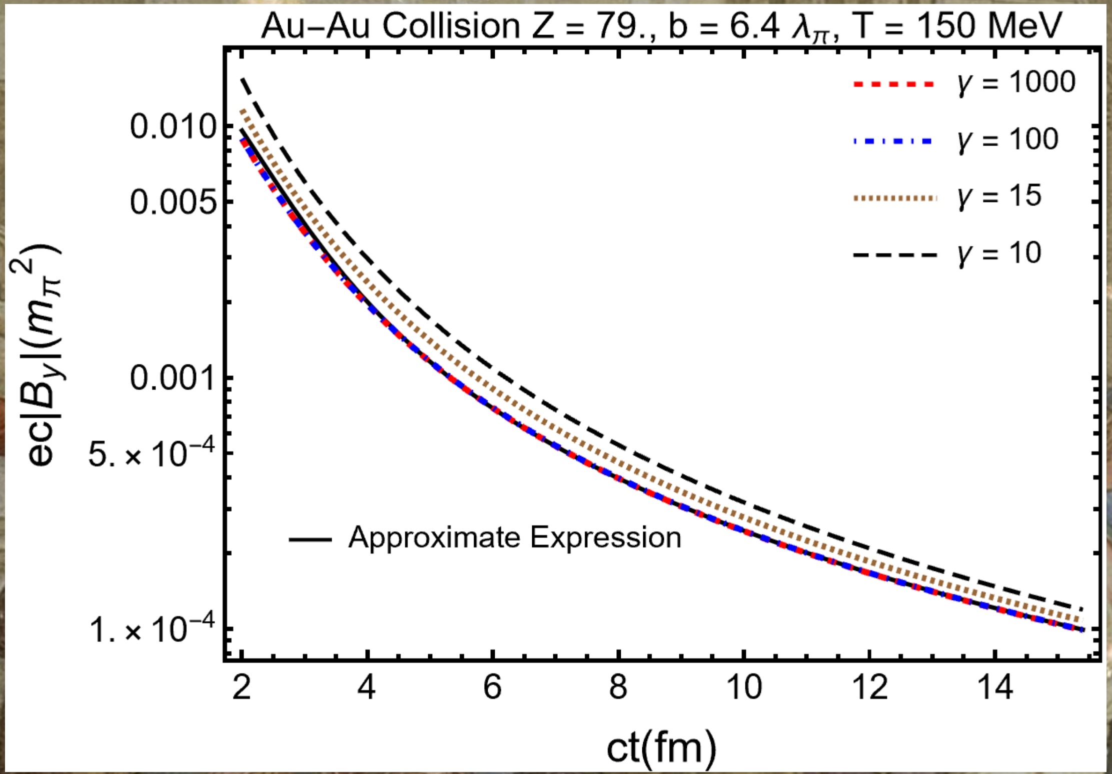
The relativistic ion field samples the polarization on the light-cone.

C. Grayson, M. Formanek, B. Müller, J. Rafelski, "EM Polarization of Quark-Gluon Plasma" In preparation. (2021)

M. Formanek, C. Grayson, J. Rafelski, B. Müller, "Current-Conserving Relativistic Linear Response for Collisional Plasmas" Annals of Physics 434 (2021) doi:10.1016/j.aop.2021.168605 [arXiv:2105.07897]

K. Tuchin, "Particle production in strong electromagnetic fields in relativistic heavy-ion collisions." Advances in High Energy Physics (2013)

At QGP-Hadronization transition, what is the magnetic field?
Freeze-out magnetic field



As the QGP fireball dissipates leaving behind hadrons, the magnetic field of the QGP remains.

$$B_y(t) = -\mu_0 \frac{Zq\beta}{2\pi} \frac{b\kappa\sigma_{static}\left(1 - i\kappa t E(\kappa t) e^{\kappa t}\right)}{4t}$$

C. Grayson, M. Formanek, B. Müller, J. Rafelski, "EM Polarization of Quark-Gluon Plasma" In preparation. (2021)

M. Formanek, C. Grayson, J. Rafelski, B. Müller, "Current-Conserving Relativistic Linear Response for Collisional Plasmas" Annals of Physics 434 (2021) doi:10.1016/j.aop.2021.168605 [arXiv:2105.07897]

K. Tuchin, "Particle production in strong electromagnetic fields in relativistic heavy-ion collisions." Advances in High Energy Physics (2013)

Let’s celebrate fifty years of published work in strong fields and SR

VOLUME 27, NUMBER 14 PHYSICAL REVIEW LETTERS 4 OCTOBER 1971

Superheavy Elements and an Upper Limit to the Electric Field Strength*

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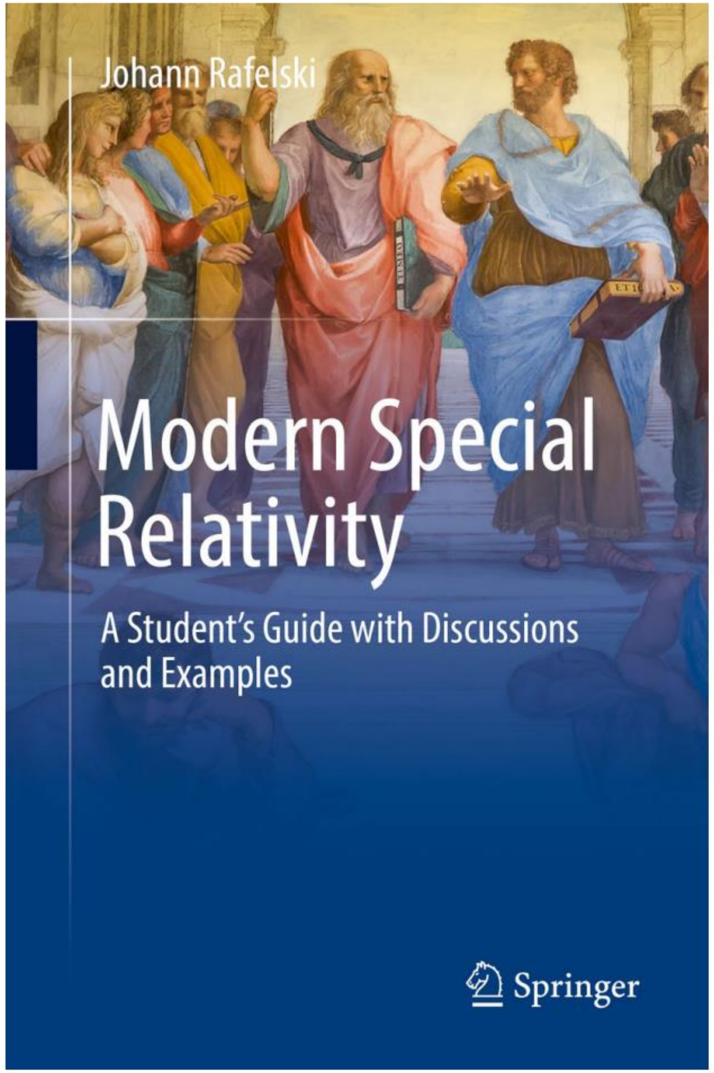
Institut für Theoretische Physik der Universität Frankfurt, Frankfurt am Main, Germany
(Received 9 August 1971)

An upper limit to the electric field strength, such as that of the nonlinear electrodynamics of Born and Infeld, leads to dramatic differences in the energy eigenvalues and wave functions of atomic electrons bound to superheavy nuclei. For example, the $1s_{1/2}$ energy level joins the lower continuum at $Z = 215$ instead of $Z = 174$, the value obtained when Maxwell's equations are used to determine the electric field.

The strong acceleration frontier requires understanding of special relativity.

Special relativity is subtle and often mixed up with spacetime and gravity.

Students are self-learners and have to warned not to rely on the internet or popular books which are for the most part written by dilettantes.



J. Rafelski

Modern Special Relativity

A Student's Guide with Discussions and Examples

- Designed for students, it provides many solved problems that help master the subject
- The concepts of special relativity are introduced in a clear and simple way, without making use of four-vector formalism
- Presents the existing connections between special relativity and particle, nuclear, and high intensity pulsed laser physics

Supplementary Slides

Internet Misconceptions:
Reinventing Lorentz-FitzGerald body contraction.

- **Many prophets claim <u>space</u> is contracted. NO!**

  Before GR (gravity!) nobody would confound the properties of a material body with space-time. SR is flat space-time for everyone.

- **Others say this is <u>distance</u> contraction. What does that mean?**

  No free ride! Either causes confusion or leads back to body contraction. A coordinate transformation (measured by physical clocks and rulers) must be consistent with the behavior of material bodies. Coordinate transformation of the body ends, measured at equal time in observers frame, is consistent with the Lorentz-FitzGerald body contraction.

- **Some claim this is an <u>apparent</u> and <u>unphysical</u> contraction. !?!?!?**

  Einstein wrote in 1911 explaining that his and Lorentz's views agree: Body contraction is physical and real.

Lorentz-FitzGerald body contraction:
What is contracted?

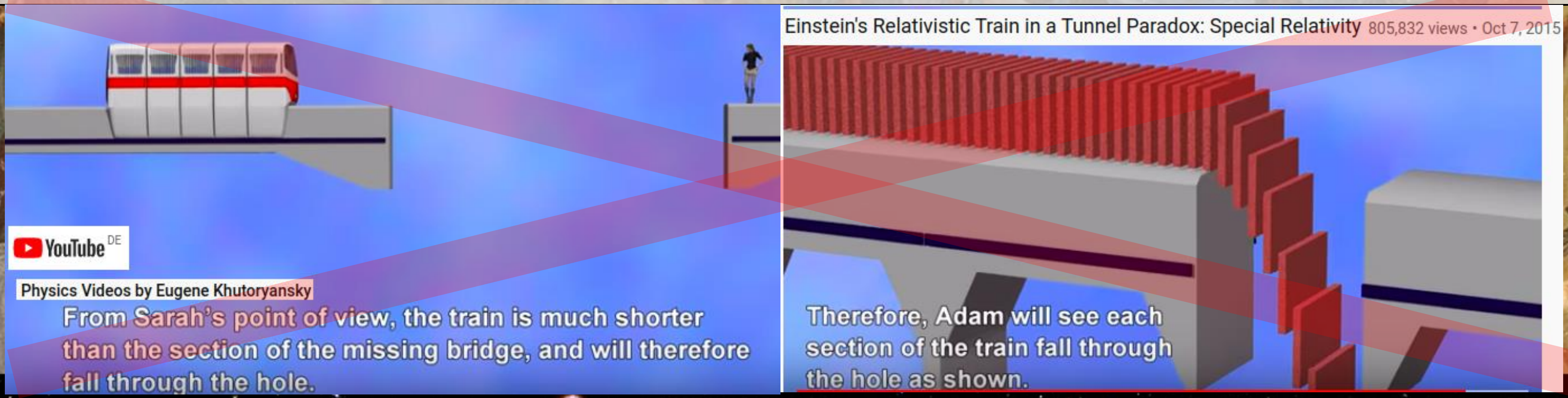
Space is NOT contracted. SR does not address the properties of space-time. The fact that different observers measure using different coordinates does not mean there is a change to space-time manifold. Riemann curvature is ZERO.

“Distance contraction” can cause unnecessary complications. Using the phrase “distance contraction” to discuss Lorentz transforms can confuse students into believing that space and time are impacted when they are not. SR is a theory of the motion of bodies only.

Lorentz-FitzGerald body contraction:
Can you see me contracted?

A measurement of body length is always carried out at an observer's equal time. A mountain tunnel and a train (at rest of moving) are measured from a station in this way. The moving train is then contracted in the direction of motion and will fit in the tunnel (or fall in the break in the bridge).

A "contraction reversal paradox" (??) does not exist. A passenger on the train measured length at her equal time. Whatever her measurement, this is not the same as station time and does not change the fact that the station observer sees the train fit in the tunnel. Everything is consistent allowing for a DIFFERENCE IN TIME.

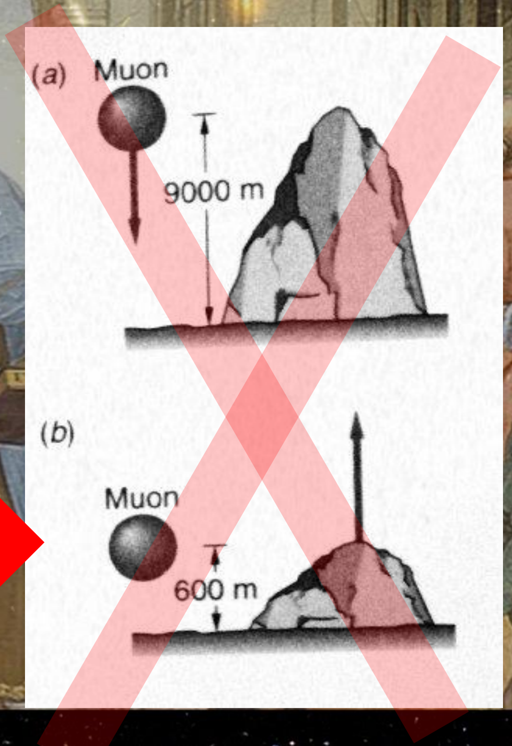


Lorentz-FitzGerald body contraction:
Are body contraction and time dilation equivalent? **NO.**

In SR, both Lorentz-FitzGerald (LFG) body contraction, and time dilation are unrelated body property phenomena (unlike energy and moment which are related). As a simple proof: Remember that **elementary point particles** can experience time dilation, but **cannot experience LFG body contraction.**

Footnote: A finite sized body is often introduced when discussing time dilation to claim equivalence with body contraction. This is a non sequitur and can lead to bad textbook pictures such as this one.

NO!!



Extended bodies in SR

- In SR, we strive to comprehend what happens to extended material bodies. LFG body contraction emerges as a pivotal concept.

- A cohesive extended body is naturally different from a cloud of non-interacting particles. Since space does not contract, a free particle cloud does not either (assuming density below some interaction range).

- All cohesive material bodies are contracted.

- Intermediate cases can still be understood in SR. See “Bell Rockets.”

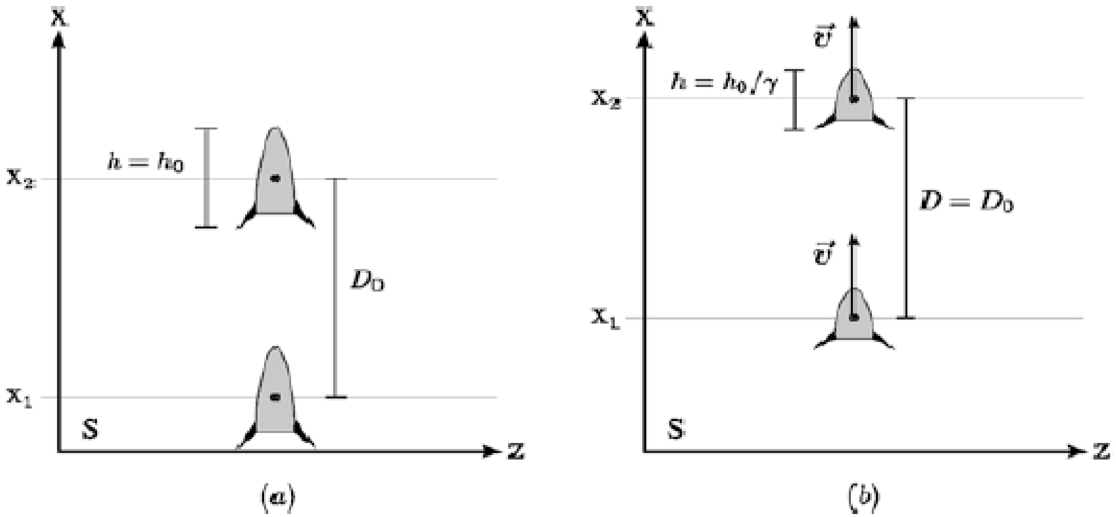


Fig. 10.2 Two rockets of length h separated by distance $D = x_2 - x_1 = D_0$. (a) at rest, and in case (b) moving at velocity $\vec{v}$ acquired at a later time

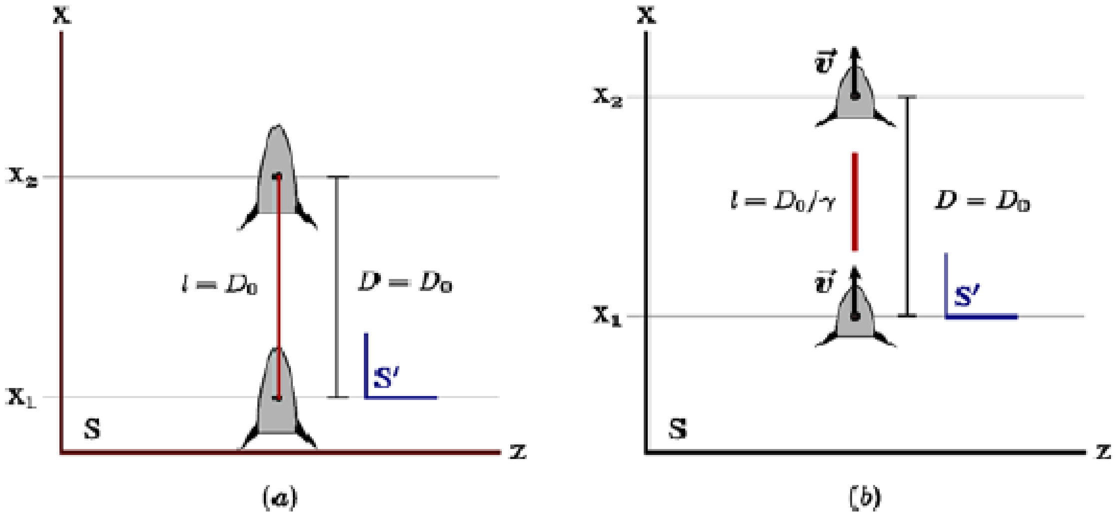


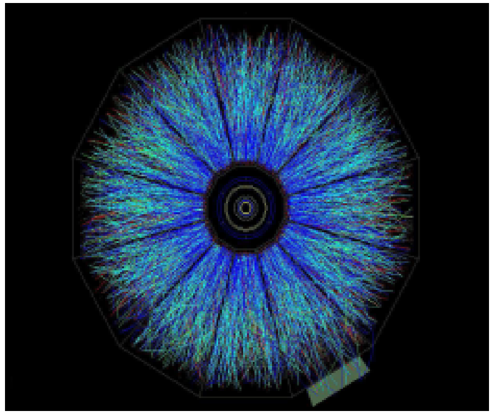
Fig. 10.3 Two rockets separated by distance $D = x_2 - x_1 = D_0$ and connected by a thin thread of (a) at rest, and in case (b) moving at velocity $\vec{v}$ acquired at a later time

Bell rockets: Spatial distance versus body length

The reality of LFG body contraction can be seen in the breaking of the thread connecting two rockets as they independently accelerate from one inertial frame to another.

The spatial distance between the rocks however is unchanged.

Unit Acceleration in Strong Interactions



Two nuclei smashed into each other at highest achievable energy: components can be stopped in CM frame within $\Delta\tau \simeq 1$ fm/c. Tracks show multitude of particles produced, as seen at RHIC (BNL) and at CERN.

- The acceleration a required to stop some/any of the components of the colliding nuclei in CM: $a \simeq \frac{\Delta y}{M_i \Delta\tau}$. Full stopping: $\Delta y_{\rm SPS} = 2.9$, $\Delta y_{\rm RHIC} = 5.4$, larger at CERN. Considering constituent quark masses $M_i \simeq M_N/3 \simeq 310$ MeV we need $\Delta\tau_{\rm SPS} < 1.8$ fm/c and longer times at colliders to exceed critical a.

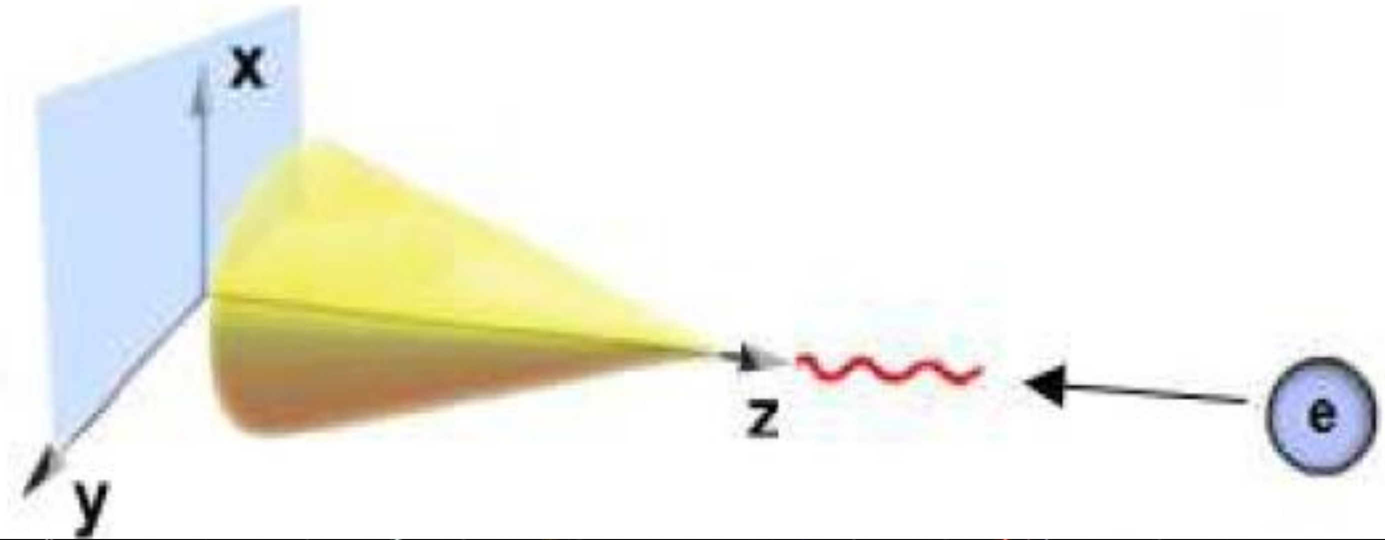
- The soft electromagnetic radiation in hadron reactions (*A. Belognni et al. [WA91 Collaboration], "Confirmation of a soft photon signal in excess of QED expectations in π–p interactions at 280-GeV/c," Phys. Lett. B* **408***, 487 (1997) [arXiv:hep-ex/9710006].*) and heavy ion reactions exceeds the perturbative QED predictions significantly

Probing super-critical (Planck) acceleration

$$a_c = 1 (\to m_e c^3/\hbar = 2.331 \times 10^{29}\mathrm{m/s^2})$$

Plan A: Directly laser accelerate electrons from rest, requires Schwinger scale field and may not be realizable – backreaction and far beyond today's laser pulse intensity technology.

Plan B: Ultra-relativistic Lorentz-boost: we collide counter-propagating electron and laser pulse.



Proposed radiation friction force
(We do not believe this is right!)

In the Lorentz-Abraham-Dirac (LAD) framework:

$$m\dot{u}^{\mu} = eF^{\mu\nu}u_{\nu} + m\tau_0\left(g^{\mu\nu} - \frac{u^{\mu}u^{\nu}}{c^2}\right)\ddot{u}_{\nu}$$

So called “Schott term” added ad-hoc to ensure:

$$u^2 = c^2$$

- Introduces intrinsic higher order derivatives.
- Requires additional initial conditions.
- Causes “runaway” solutions.
- Causality issues for small times.

Sources radiated power from Liénard-Wiechert solution of Maxwell’s equations.

$$P = m\tau_0\dot{u}^2$$

The time-scale $\boldsymbol{\tau_0}$ is

$$\tau_0 = \frac{2}{3}\frac{e^2}{4\pi\epsilon_0 mc^3} = 6.3\times 10^{-24}\ \mathrm{s}$$

Liénard-Wiechert (LW) field of a moving charge

Each point particle in an ion contributes to the overall field with a LW field:

$$e\boldsymbol{E}(\boldsymbol{r},t) = Z\alpha\hbar c\left(\frac{(\boldsymbol{n}-\boldsymbol{\beta})}{\gamma^2(1-\boldsymbol{n}\cdot\boldsymbol{\beta})^3|\boldsymbol{r}-\boldsymbol{r_s}|^2} + \frac{\boldsymbol{n}\times((\boldsymbol{n}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}})}{c(1-\boldsymbol{n}\cdot\boldsymbol{\beta})^3|\boldsymbol{r}-\boldsymbol{r_s}|}\right)_{t_r}$$

$$e\boldsymbol{B}(\boldsymbol{r},t) = \frac{\boldsymbol{n}(t_r)}{c}\times\boldsymbol{E}(\boldsymbol{r},t) \qquad \text{where} \qquad t_r + \frac{1}{c}|\boldsymbol{r}-\boldsymbol{r_s}(t_r)| = t$$

- LW field: Fields of an arbitrarily moving relativistic point particle derived assuming a current density.
- Often it is assumed that ions travel in straight line motion, or that $\dot{\boldsymbol{\beta}} = 0$ which is not always a good argument to neglect the acceleration term in the field.
- When acceleration is strong, radiation field dominates the velocity field, and **it radiates energy.**

Radiation reaction and limiting acceleration

(Price, Formanek, Rafelski, PRD 105 016024 (2022))

LL is lowest order weak field approximation of EFO

$$\tau_0 = \frac{2}{3}\frac{e^2}{4\pi\epsilon_0 mc^3}$$

Name	Covariant equation	Year
Lorentz-Abraham-Dirac (LAD)	$ma^\mu = \mathcal{F}^\mu + \tau_0 P^\mu_\nu \frac{d}{d\tau}(ma^\nu)$	1938
Eliezer-Ford-O'Connell (EFO)	$ma^\mu = \mathcal{F}^\mu + \tau_0 P^\mu_\nu \frac{d}{d\tau}(eF^{\nu\alpha}u_\alpha)$	1948, 1991
Landau-Lifshitz (LL)	$ma^\mu = \mathcal{F}^\mu + \tau_0(e\frac{d}{d\tau}(F^{\mu\nu})u_\nu + \frac{e^2}{m}P^\mu_\nu F^{\nu\alpha}F_{\alpha\beta}u^\beta)$	1962
Mo-Papas (MP)	$ma^\mu = \mathcal{F}^\mu + e\tau_0 P^\mu_\nu F^{\nu\alpha}a_\alpha$	1971

$$P^{\mu\nu} = g^{\mu\nu} - \frac{u^\mu u^\nu}{c^2}$$

$$a_{RR} \equiv \frac{c}{\tau_0} = \frac{3}{2\alpha}a_{\rm QED} = 4.78 \times \frac{10^{31}\text{m}}{\text{s}^2}\ \text{(electrons)} \quad a_{QED} = \frac{eE_{QED}}{m} = \frac{mc^3}{\hbar}$$

"Schwinger critical acceleration"

In constant homogeneous fields EFO = MP gives:

$$a^2 = a_{LF}^2 \frac{1 + \tau_0^2 \frac{e^4}{m^4}\frac{c^2\mathcal{P}^2}{|a_{LF}^2|}}{1 + \tau_0^2\left(\frac{e^2}{m^2}2\mathcal{S} + \frac{|a_{LF}^2|}{c^2}\right)} \xrightarrow{\text{For strong fields}} -\frac{c^2}{\tau_0^2}$$

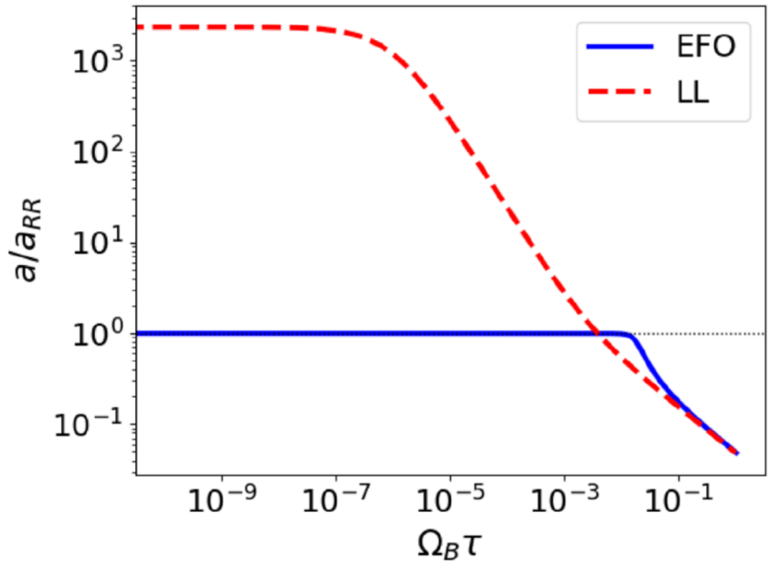
Acceleration has a Lorentz invariant upper limit in certain strong field configurations – pure magnetic field, electron turning into an electric field and more.

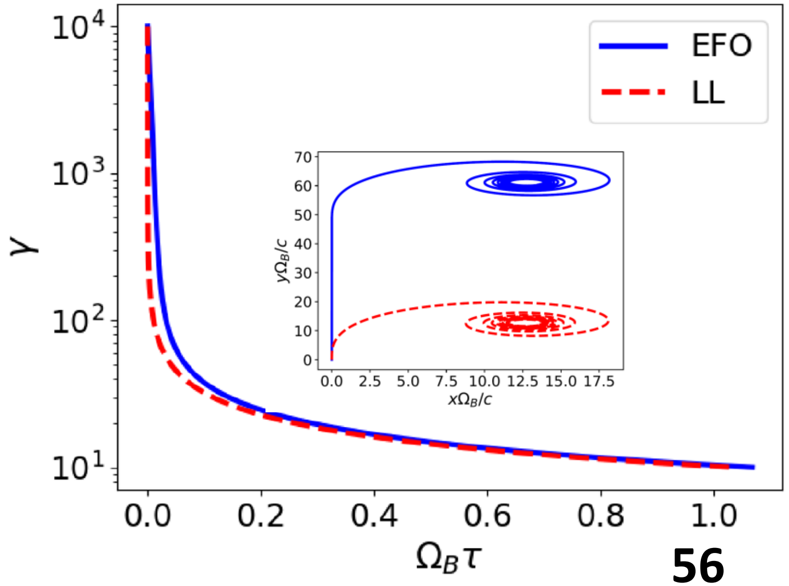
Limiting RR acceleration and limiting Larmor radiation rate:

$$P_{RR} = -\frac{mc^2}{\tau_0}$$

In pure magnetic fields ($B = 4.4 \cdot 10^9$ T):

$$\Omega_B = \frac{eB}{m} \qquad a = \sqrt{|a^2|}$$





Quark-gluon plasma (QGP) *probed in heavy-ion collisions*
Visualization of EM fields in relativistic collisions

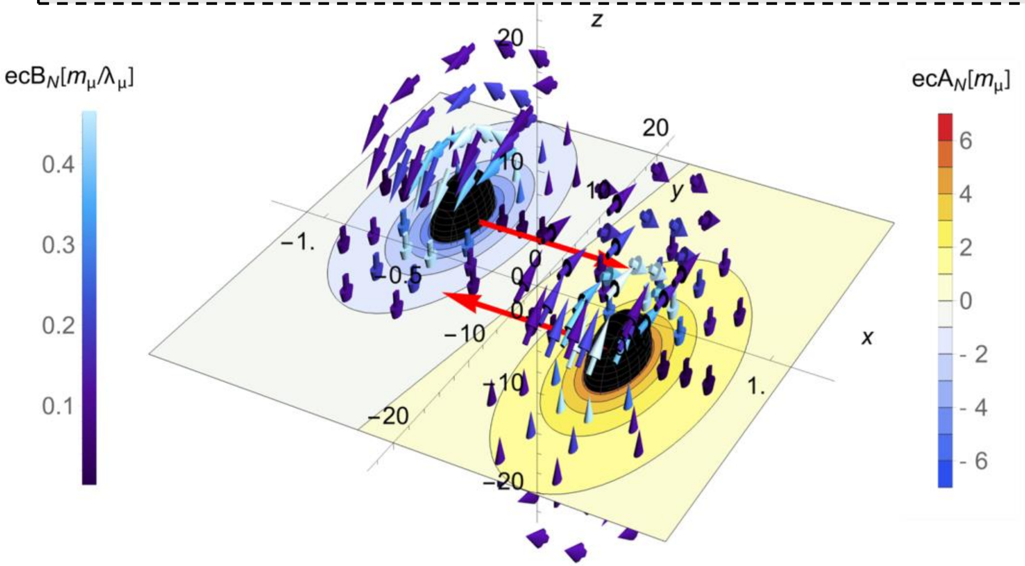
$$\lambda_\mu = \frac{h}{m_\mu c}$$

The natural EM fields $F^{\mu\nu}_{ext}$ of the ions is described used Lienard-Wiechert fields, which contains a boosted Coulomb field proportional to velocity and an acceleration or radiation field.

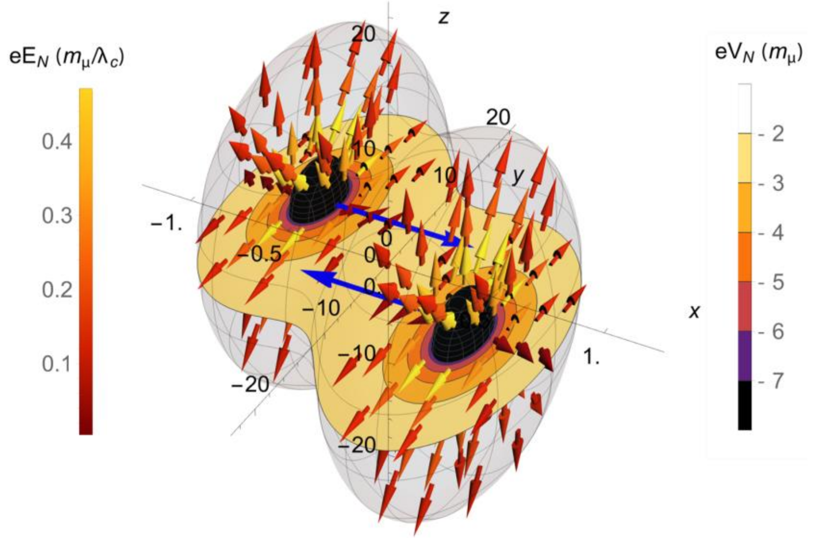
$$e\boldsymbol{E}(\boldsymbol{r},t) = Z\alpha\hbar c\left(\frac{\boldsymbol{n}-\boldsymbol{\beta}}{\gamma^2(1-\boldsymbol{n}\cdot\boldsymbol{\beta})^3|\boldsymbol{r}-\boldsymbol{r_s}|^2} + \frac{\boldsymbol{n}\times\left((\boldsymbol{n}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}\right)}{c(1-\boldsymbol{n}\cdot\boldsymbol{\beta})^3|\boldsymbol{r}-\boldsymbol{r_s}|}\right)_{t_r}$$

Velocity Field ***Acceleration Field***

Ions in the center-of-momentum frame are truly relativistic pancakes.
Here, we are simulating Pb-Pb collisions with a Lorentz factor of $\gamma = 37$.



Magnetic Field



Electric Field

Original scale